Adding It All Up

In this lesson, students draw various polygons and investigate their interior angles. The investigation is done using both an applet and paper and pencil to foster an understanding of how different patterns can lead to the same solution. After comparing results with a partner, students develop a formula showing the relationship between the number of sides of a polygon and the sum of the interior angles.

Learning Objectives

Students will:

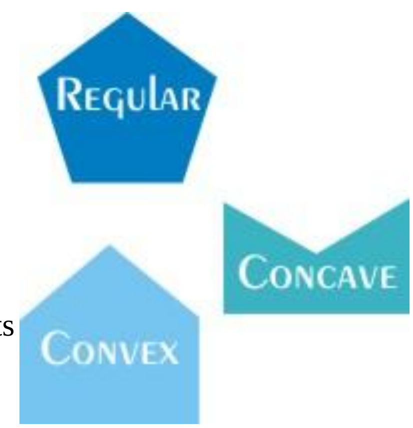
- Investigate the pattern between the number of sides of a polygon and the sum of the interior angles using in two different methods
- Determine that the interior angle sum is always the same for polygons with the same number of sides
- Create a formula to find the interior angle sum given the number of sides
- Explore interior angles in regular polygons

Materials

- Unlined paper
- Rulers
- Colored pencils or markers (optional)
- Computers with Internet access
- Angle Sum Applet
- Adding It All Up Activity Sheet
- Adding It All Up Answer Key

Instructional Plan

Begin the lesson, by reviewing key vocabulary with students:



- *Polygon:* a closed plane figure made of line segments
- *Convex:* the measures of all interior angles are less than 180°
- *Concave:* the measure of at least one interior angle is greater than 180°
- *Regular:* all angles and sides are congruent
- *Triangulation:* the process of drawing diagonals (segments between non-adjacent vertices) to divide a polygon into non-overlapping triangles

Tell students that they will be working with polygons. It is up to them whether they use regular or non-regular polygons, and whether they use convex or concave polygons. Encourage students to draw different kinds of polygons. For example, when exploring pentagons, point out that all of the examples on the right are polygons.

Tell students that they will use the Angle Sum applet to draw polygons with different angle measures to see what happens to the angles and the angle sum. Then, they will use a process called triangulation to help explain their results.

Explain to students that all polygons can be broken up into non-overlapping triangles. Show an example such as the one below, drawing diagonals to create the triangles. Explain that this process is called *triangulation*. Tell them that the triangles can help them find the sum of the interior angles of any polygon. After they find the interior angle sum and triangulation for several different polygons, they will find a formula for the interior angle sum that applies to all polygons.

Pass out the Adding It All Up activity sheet to each student. You may choose to have students work in pairs of mixed ability so they can help each other through the activity. However, each students should complete an activity sheet.

Read the directions on the activity sheet with students and direct them to first determine how many sides are in each polygon listed. Tell them that although they are working in pairs, each student must draw at least one of each polygon and triangulate it on their own, so that they have data for multiple samples of each polygon. Remind students that the polygons do not need to be regular or convex. Encourage students to try polygons that are not regular and concave. You may wish to offer students colored pencils or markers for the triangulation. This may make it easier for students to make sure the diagonals are non-overlapping.

As students work, circulate to ensure that they are drawing multiple polygons of each type and triangulating correctly. Emphasize that the sides of the triangles must be diagonals (segments connecting non-adjacent vertices) and the diagonals cannot intersect. Once they finish working with the applet and drawing their own polygons and triangulations, students should work in pairs to answer the questions on the activity sheet.

Students may struggle with finding the formula $180(n - 2)$, but do not tell them the pattern. Instead, encourage them with questions that lead toward the solution. You may want to begin a discussion about triangles. For example:

- How many triangles were you able to draw in that polygon? How does that number relate to the number of sides?

[For any polygon, there will be 2 fewer triangles than the number of sides. For example, a hexagon has 6 sides, so a triangulated hexagon is made of 4 triangles.]

- Can you triangulate a triangle?

  [No. A triangle has no diagonals.]

- What is the sum of the angles in 1 triangle? in 2 triangles?

  [The sum is 180° in a single triangle and 360° in 2 triangles because 180 + 180 = 360. Encourage students to explore this pattern and discover how it applies to triangulated polygons.]

Encourage students to ask themselves the same questions as they explore each polygon. Do they see a pattern? What happens to the number of triangles as the number of sides increases? What happens to the sum of the interior angles as the number of sides or triangles increases?

Question 3 will be especially challenging to some students. Once students recognize the pattern, they may still have difficulty expressing it algebraically. Help students by asking what the variable *n* will represent in the formula. If they try to use a variable for number of triangles, ask them how they can relate that back to the *n*-gon, which has *n* sides. Remind students to ask themselves, How can I find the number of triangles if I know the number of sides.

Once students have had sufficient time to find the formula, walk through the process of finding it as a group (the formula and the other answers can be found on the Adding It All Up answer key). This will help those who were not able to find the formula themselves to catch up. Ask students if the formula works every time. Check it as a class using some of the polygons from their chart and discuss any other patterns students may have discovered during the exploration.

Questions for Students

- Are all the angle measures always the same for a single polygon?

  [No. If all the angles are congruent, the polygon is called *equiangular*.]

- As the number of sides increases, what happens to the sum of the angle measures?

  [For each additional side in a polygon, 180° is added to the sum of the angle measures.]

- Does the formula work for both regular and non-regular polygons? What about shapes like scalene triangles or trapezoids?

[Yes, the formula works for all polygons.]

Assessment Options

1. Have students write a journal entry describing how to find the interior angle sum of any polygon. Students should include details such as patterns they discovered and other questions on the topic they would like to explore.
2. Create a set of cards numbered from 50 to 100. Randomly give 1 card to each student. Ask all students to find the sum of the interior angles of a polygon with the number of sides shown on their card and the measure of 1 interior angle of a regular polygon with that same number of sides.

Extensions

1. Have students explore exterior angles. The sum of the exterior angles for any polygon is 360°, and therefore the measure of one exterior angle in a regular polygon is $^{360}/_{n}$.
2. In the bottom right-hand corner of the Angle Sum applet, there is an animation for the triangle and square showing how the sum of the interior angles relates to tiling. Have students watch the animations and write a journal entry on what they demonstrate.

Teacher Reflection

- Were students able to use the applet without help?
- Did students have sufficient data to develop a formula? What other data could have been provided?
- Did students work well in pairs, or would other groupings work better?
- Were students able to communicate with their classmates so that everyone had the same understanding at the end of the activity?

NCTM Standards and Expectations

Geometry 6-8

1. Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects
2. Draw geometric objects with specified properties, such as side lengths or angle measures.

Measurement 6-8

1. Develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes.

This lesson was prepared by Katie Hendrickson.