

Exploring Linear Functions

Technology allows the linking of multiple representations of mathematical situations and the exploration of the relationships that emerge. This example presents a series of explorations based on two linked representations of linear functions. The grades 9–12 section on the [Problem Solving Standard](#) includes an episode describing how a teacher engaged her students in problem solving and reasoning with tasks such as those presented in this example.

Learning Objectives

- use technology to link multiple representations of mathematical situations
- explore of the relationships

Materials

- Computer and Internet connection - <http://illuminations.nctm.org/LessonDetail.aspx?ID=L492>

Instructional Plan

Tasks

1. How do changes in the values of m and b in the linear function $f(x) = mx + b$ affect the graph? In the applet below, use the sliders to adjust m and b in $f(x) = mx + b$. How is the graph affected by changes in m ? By changes in b ?
2. What are the effects on the graph of $f(x) = mx + b$ if the values of m and b are changed simultaneously? Click on the Connect Sliders button. Adjust the sliders again. What happens to the equation and to the graph? Summarize your findings.
3. What are the effects of changing m and b simultaneously for different lines? Uncheck the Connect Sliders button, and specify a different initial line. Then click on Connect Sliders again. Describe what happens. Compare your results to the results you obtained in task 2.
4. Click on the Show Trace button. Predict starting values for m and b that, with the sliders connected, would result in the lines intersecting in—

Quadrant I,
Quadrant II,
Quadrant III,
Quadrant IV.

Try out your values. Describe what happens.

5. If m and b are simultaneously varied by the same amount, a family of lines is generated.
- Express the equation $f(x) = mx + b$ so that it represents the family of lines. (*Hint: Consider using k to represent the amount m and b are varied.*)
 - Use your equation to explain why all the lines meet at a common point and why the x -value at that common point is always -1 .

Discussion

Dynamic graphical representations can suggest mathematical relationships that may be surprising to students and teachers, and exploring such relationships can help students gain a deeper understanding of important concepts. With technology, representations can be manipulated rapidly and in ways that would not be practical using paper and a pencil.

The purpose of this task is for students to better understand linear functions by exploring the relationship between symbolic and graphical representations. The first task draws students' attention to the roles played by the parameters in a linear function and engages them in observing, describing, and comparing relationships among mathematical objects. They can also begin to link representations.

In addition to exploring what happens when b and m are adjusted separately, students can use the Connect Sliders option to explore what happens when they simultaneously vary b and m by the same amount. In trying to understand why any pair of lines seems to intersect at a common point, students will find that it is useful to develop a general symbolic representation of the family of lines whose slopes and intercepts differ by the same amount. They can then prove that all such lines will pass through the point $(-1, b - m)$.

The episode on pages 338–40 in the [Problem Solving](#) section for grades 9–12 describes how this situation developed in an instructional setting. See that discussion for further details.

Take Time to Reflect

- What do you think are students' greatest conceptual difficulties in representing graphical relationships with symbols?
- What questions would you ask students in order to better understand their thinking about the relationships they see?
- What probing questions could you ask to help students expand their understanding of graphical and symbolic representations and the relationship between them?

- How might you help students see the need to explain more formally the phenomena they are observing?

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