Building Connections

This lesson focuses on having students make connections among different classes of polynomial functions by exploring the graphs of the functions. The questions in the activity sheets allow students to make connections between the x-intercepts of the graph of a polynomial and the polynomial's factors. This activity is designed for students who already have a strong understanding of linear functions, some knowledge of quadratic functions, and what is meant by a polynomial function.

NGSSS

MA.8.A.1.5 – Representations of Linear Functions
MA.8.A.1.6 – Compare graphs of linear and nonlinear functions

Learning Objectives

- Explain the relationship between linear factors of a polynomial function and the graph of the function
- Based on the graph of two lines, sketch the parabola that is the product of the two linear expressions
- Given the graph of a polynomial, find the equations of lines that could be components of the polynomial

Materials

- Colored pencils
- Strips of paper or rulers
- Graphing calculators (optional)
- Building Polynomial Functions Activity Sheet
- Working Backwards Activity Sheet
- Higher Degree Polynomials Activity Sheet

Instructional Plan

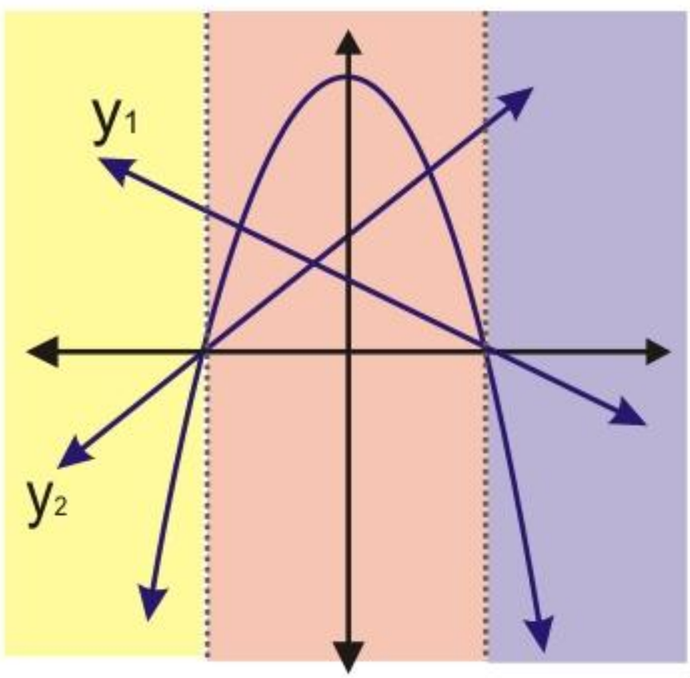
Encourage students to work in pairs on the activity sheets. The discussion generated by questions in the activity is beneficial. Each student needs the activity sheets, three different colored pencils, and a strip of paper or a ruler. The activity is appropriate in either first- or second-year algebra, as soon as students have a good foundation in linear functions, some knowledge of quadratic functions, and an understanding of what is meant by a polynomial function.

Building Polynomial Functions

Students start by identifying a linear function and putting the equation in slope/*x*-intercept form, $y = m(x - c)$, where c is the *x*-intercept. This form serves as a connector with other classes of polynomial functions and forces students to focus on the *x*-intercept of the graph. They then choose another function in the form $y = m(x - c)$ and graph this function on the same axes. Students predict how a new function, formed by taking the product of the two linear expressions, would appear graphically. After making their prediction, they graph the resulting quadratic function and compare the actual function with their prediction. Students can use a graphing utility to check the function formed by taking the product of the linear factors, but only after making the prediction.

Activity questions that compare the linear functions with the resulting quadratic function focus the students' attention on the parts of the graphs to be emphasized. Students learn that the quadratic function has the same *x*-intercepts as the linear functions, which can be quite a revelation, and that the *y*-intercept of the quadratic function is the product of the y-intercepts of the linear functions. In fact, the *y*-coordinate of the parabola for a given *x*-value is always the product of the *y*-coordinates of the lines for that *x*-value. Seeing this relationship is easier when *x* equals 0 and the *y*-coordinates are lined up on the *y*-axis.

Students then use a strip of paper or a ruler to cover parts of the graph. This part of the activity shows that the sign of the *y*-coordinate for any point on the parabola can be determined by observing whether the *y*-coordinates of the lines for that section of the graph are positive or negative. For example, if both lines in a section of the graph are above the *x*-axis, then the parabola will be above the *x*-axis, that is, (+) • (+) = (+). If one line in a section of the graph is above the *x*-axis and the other is below the *x*-axis, then the parabola is below the *x*-axis, that is, (+) • (–) = (–). This result corresponds to the sign table that students have traditionally used as an aid to graph functions and inequalities.



Working Backwards

The second part of the activity requires students to work in the opposite direction, that is, take a graph of a quadratic function and break it into its linear components. Classroom experiences often focus on factoring quadratic expressions into linear factors. The graphical counterpart to this process is to break the graph of a quadratic function into its components, the lines. This illuminates factoring in a visual way. You should caution students that a graph that appears to be a parabola may actually be the graph of a fourth-degree or higher even-degree polynomial function. Beyond this activity, students should not assume that a graph that has two *x*-intercepts is the graph of a second-degree polynomial function.

When students first attempt this activity, they usually focus on having their lines go through the parabola's *x*-intercepts, but they may not consider the *y*-intercepts. They should also check sections of the graphs before and after the *x*-intercepts to make sure that the product of the *y*-coordinates of the linear factors gives the sign and values of the *y*-coordinates on the parabola in that section.

When asked whether the choice of these lines is unique (it is not), students may have some conflict. They have been told that quadratic expressions factor uniquely into linear expressions; however, they may not have been told that this outcome is unique only up to unit factors. Unit factors can therefore be split over linear expressions to give an infinite number of combinations of lines; for example, $(2x - 3) \bullet (x + 4)$ is equivalent to $(1/2)(2x - 3) \bullet 2(x + 4) = (x - (3/2)(2x + 8)$. The only limitations for possible lines are that the product of the *y*-intercept for the parabola and the *x*-intercepts must remain the same.

The last graph in this section has no *x*-intercepts, but students are asked to try to sketch lines that could be its components. This example should prompt some good discussion. Students should eventually conclude that an absence of *x*-intercepts implies that no real roots exist - that is, lines cannot be drawn because the quadratic equation cannot be factored into linear expressions over the real numbers. This visual display fosters insight into why quadratic equations sometimes cannot be factored in the real-number system.

Higher Degree Polynomials

When students have experience building quadratic functions from linear expressions and working backward to find linear components, extending these ideas to polynomials of degree three is a natural progression. The teacher should warn students that graphs of fifth-degree or higher odd-degree polynomials can closely resemble the graph of a cubic polynomial. Therefore, beyond this activity, they should not assume that a graph is a cubic because it has three *x*-intercepts.

Summarizing the lesson

After students have completed this activity, the teacher should have the class discuss the material. Students should share their insights and ask questions.

This new way of looking at polynomial functions enriches and broadens students' conceptions about polynomial functions. Visualizing the algebraic representation through a graph gives more meaning to the symbols. These activities should make the connections between the *x*-intercept and the factors of the polynomial more salient, as well as highlight for students "the glue" that holds the classes of polynomial functions together.

Assessment Option

1. A suggestion for assessment is to have students summarize the activity by writing about what they have learned and discussing their understanding of the relationships among linear factors, polynomial functions, and the graphs of these functions. Their comments shed insights into their thinking and help suggest improvements in the activity.
2. A quiz might consist of having the students graph two lines and then sketch the parabola that represents the product of the two linear expressions. Make sure that students explain their reasoning. They could also be instructed to work backward - given the graph of a parabola, they could sketch and find the equations of possible lines that are components of that parabola.

   I have found that students tend to blur the terms *factor* and *x-intercept*. In assessing their writing and oral communication, the teacher should insist that students use the proper terminology and should clarify any "fuzziness" that may exist between these two concepts

Teacher Reflection

- What activities would (*a*) foster connections among the classes of polynomial functions, that is, linear functions, quadratic functions, and polynomial functions of degree greater than two, and (*b*) foster connections between the graphical and algebraic representations of these functions?
- What connections do the students make between their study of the graphs of linear and quadratic functions and their study of the graphs of polynomial functions of degree greater than two?
- Does their understanding of polynomial functions of degree greater than two build on their understanding of the graphs of linear and quadratic functions?

References

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- Curran, Judy. "An Investigation into Studentsç Conceptual Understanding of the Graphical Representation of Polynomial Functions." Ph.S. diss., University of New Hampshire, 1995.
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