Bathtub Water Levels

This lesson is similar to Lesson One: Traveling Distances; however, this lesson is designed so students examine real-life data that illustrates a negative slope. Students interpret the meaning of the negative slope and *y*-intercept of the graph of the real-life data. By examining the graphical representation of the data, students relate the slope and *y*-intercept of the least squares regression line to the real-life data. They also interpret the correlation coefficient of the least squares regression line.

NGSSS

MA.8.A.1.5 – Representations of Linear Functions

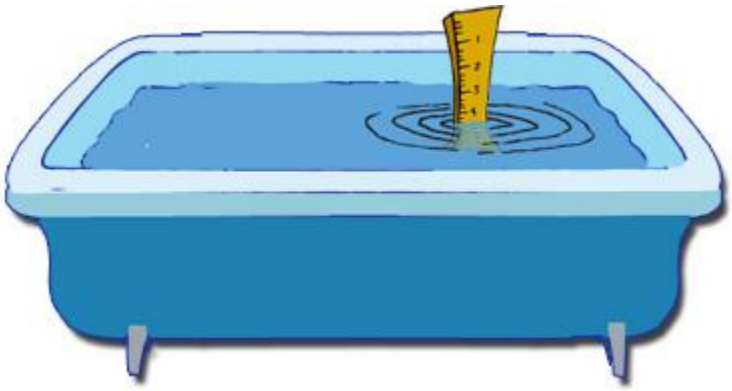
Learning Objectives

- plot data points
- interpret the negative slope of a line as a rate of change in the context of real-life data
- interpret the *y*-intercept of the line in the context of real-life data
- interpret the meaning of the correlation coefficient of the least squares regression line

Materials

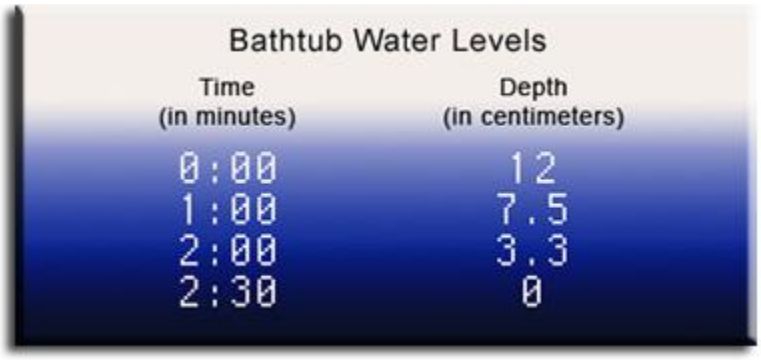
- Computer and Internet connection
- Graphing Real-Life Data recording sheet
- Status of the Class recording sheet

Instructional Plan



This lesson may be conducted in one of two methods. The teacher may provide the data needed to conduct the lesson or the students can gather the data as a homework assignment at the end of Lesson One. The data in the following table can be placed on the board if the teacher is to provide the necessary information. If the students do the data collection as a homework assignment at the end of Lesson One, they should be instructed to complete a table similar to the

following chart. They should fill the bathtub as though they were going to take a bath and measure the height of the water in centimeters. Then they should set a timer for one minute, start the timer, and open the drain plug. At the end of one minute, they should close the drain plug and measure the height of the water in centimeters. Students should measure the height of the water near the middle of the tub, away from the sides. They should record their data and continue this process until the tub is empty. The time should be measured in minutes and the depth of the water should be measured in centimeters.



Bathtub Water Levels

Time (in minutes)	Depth (in centimeters)
0:00	12
1:00	7.5
2:00	3.3
2:30	0

This lesson should begin with a discussion of slope, y-intercepts, and correlation coefficients. The discussion should include some of the topics discussed at the beginning of Lesson One and those covered in Lesson One; particular attention should be paid to the students' responses to questions 2, 3, 5, and 6 below. The answers to these questions indicate how well students understood Lesson One and will indicate to the teacher that topics need to be stressed during this lesson. The following guiding questions will help you with your discussion.

1. What is the slope of a line? What does it mean to the graph of a line?
2. What is the relationship between the slope of a line and the rate of change?
3. How do the labels on the axes of a graph help determine the rate of change?
4. What do we know about the *y*-intercept of any graph?
5. What does the *y*-intercept tell us about a real-life application?
6. What is a correlation coefficient? What does it tell us about the points being plotted and the resulting least squares regression line?

During the remainder of this lesson, the teacher should continue to help students make the connection between the slope of a line and its meaning as the rate of change in a real-life application. The meaning of the *y*-intercept in the context of the same application should also be stressed.

The teacher should go to the Web site: http://illuminations.nctm.org/index_d.aspx?id=454 and demonstrate how to change the window settings and plot the data provided in the chart or gathered by the students. The points that are plotted should be the total number of minutes the drain has been open (x) vs. the depth of the water in centimeters (y). After the points have been plotted, the teacher should stop to direct the students' focus to some of the mathematics involved in the plot.

- Do you notice any pattern in the shape of the plot?
- What type of function do you think will fit this data?
- Do you think the slope of the line will be positive or negative?
- What is the *y*-intercept of the line and what does it mean?

The teacher should now use the applet to graph the least squares regression line and calculate the correlation coefficient. Direct the students' attention to the slope of the line and ask what relationship it has to the data. The teacher should ask questions that lead the students to interpret the slope as a drop in the water level of 4.7 centimeters per minute for the data in the chart. This indicates a rate of change--it indicates how many centimeters per minute the water level is dropping. Ask what labels would be appropriate for both axes, and how this information could help them calculate the rate of change.

Next, examine the *y*-intercept of the line. When the points in the table are plotted, this applet gives a *y*-intercept of 12.4 centimeters. Ask the students what it means in the context of this problem. With effective questions, lead the students to see that the *y*-intercept in the applet means that the initial height of the water, based on the equation, would be 12.4 centimeters. Does this make sense? Explain that this is the predicted value, not the actual value. Ask how the appropriate labels on the axes can help the students interpret the meaning of the *y*-intercept in the context of this problem.

The correlation coefficient should now be addressed. Ask the students to explain what the correlation coefficient of -1 means in terms of this example. Direct the questioning to get the students to see that the line passes through the data points and the line has a negative slope. Refer to the fact that the correlation coefficient in Lesson One was positive one and the line had a positive slope. The line might not pass through all the data points, especially if students collect their own data.

The students should now graph this data on the second graph on the handout Graphing Real-Life Data that were given in Lesson One. They should be given the opportunity to label the axes, indicate the scale, plot the points, draw the line through the data, and fill in the remaining information for the second graph on the handout. Grouping the students in pairs while they do this part of the lesson will allow them to discuss what they are doing and help reinforce the ideas of the lesson.

As the students work, the teacher should circulate among the students to observe their work and answer any questions they may have at this point. After they have finished their work on the graph, the teacher and students should summarize the lesson by completing the items next to the graph on the handout. The students should then be directed to save their handout, so they may refer to it during other lessons in this Unit Plan.

Assessment Options

The discussion at the end of the graphing activity will give both the teacher and the students an opportunity to assess the students' understanding of the lesson. At this stage of the activity, it is important to know if the students can:

- Correctly plot data points by hand
- Interpret the meaning of a negative slope as a rate of change in the context of this problem
- Interpret the meaning of the *y*-intercept in the context of this problem
- Understand the meaning of a negative correlation coefficient in the context of this problem
- Interpret the meaning of the labels and units on each of the axes

Using the handout during the class discussion will help the students focus on the mathematics involved in the graph. The handout will also provide information the students can use to help them answer questions in other parts of this Unit Plan. A Teacher Resource Sheet, Status of the Class, can be used for recording information about students' understanding of this and other lessons in the Unit Plan. It is helpful to record students' current levels of understanding as a way to plan instruction and to monitor and measure their growth toward meeting the learning objectives. Documenting information about students' understanding throughout the lesson(s) can help you focus on individual student's needs and strengths, and thus can improve student learning opportunities.

The assessment information you collect can help you monitor student learning, adjust instruction, and plan future lessons for the class. Data on individual students can be used to plan strategies for regrouping students, remediation, and extension activities. This information is extremely useful when discussing progress toward learning targets with students, parents, administrators, and colleagues.

Extensions

The following questions can be used to help students relate what they have done during this lesson to other topics about linear functions.

1. For what values of *x* did your equation seem appropriate? Why?
2. What values of y do you think would be appropriate for the real-life data used in this example? Why?
3. What would be an appropriate domain for your function?
4. What would be an appropriate range for your function?

Teacher Reflection

1. Which students met all the objectives of this lesson? What extension activities are appropriate for those students?
2. Which students did not meet the objectives of this lesson? What instructional experiences do they need next? What mathematical ideas need clarification?
3. What adjustments would you make the next time you teach this lesson?