Extending to Symbols

In this investigation, students learn about the notion of equivalence in concrete and numerical settings.

As students begin to use symbolic representations, they use variables as place holders or unknowns. This investigation illustrates the continued transition from the concrete balance view of equivalence to a more abstract view.

Learning Objectives

- investigate the equivalence of two symbolic expressions
- transition from numeric to algebraic notation

Materials

- Pan Balance - Expressions

Instructional Plan

In this investigation, students learn about the notion of equivalence in concrete and numerical settings.

As students begin to use symbolic representations they use variables as place holders or unknowns. As their use of symbolic representation grows, they need to engage in reasoning about the symbolic representation. For example, in addition to experience with equivalent representations that arise from a contextual situation, students should explore questions which arise within the symbolic representation such as:

- Under what conditions are two symbolic expressions equal?
- When is an equation an identity?
- What operations within the algebraic system preserve solutions or equality?

One method of investigating these questions is through the connection of symbolic representations to other representations such as tables or graphs. This investigation illustrates the continued transition from the concrete balance view of equivalence to a more abstract view.

Try This Task! Pan Balance - Expressions

Use the interactive tool above to explore the following questions. Use the following symbols:

- + for addition
- - for subtraction
- * for multiplication
- / for division
- ^ to raise an expression to a power

Students should perform the following steps and answer the questions which follow.

Enter the expressions x + x and 2*x into the boxes on the pans.

Change the value of x using the slider or by clicking and holding the mouse button down while dragging the cursor in the graph window.

> What do you notice as you change the value of x?
> What do you notice about the behavior of the balance when you use the two expressions 7 – x and x – 7? What does this behavior correspond to in the graph?
> What is an expression which is never equal to 7 – x? How do you know from the graph that these expressions are never equal?

Think About These Situations

For each of the following sequences, find two expressions such that as x increases, the position of the balance moves according to the sequence.

For example, in (i) / - \ , find two expressions so that when your cursor is at the left edge of the graph the left side is down (/), then ,as you move your cursor to the right, there is a place where the sides are balanced (-), and then, as you continue to move your cursor to the right, the right side goes down (\).

(i). / – \ (ii). \ – / – \ (iii). \ / (Never balanced)

Use the pan balance to test expressions that you think might equal (balance) the following expressions. Use ^ to raise a number, variable, or expression to a power.

> x * (x + 1)
> (x + 1) / x
> x / (x + 1)
> (x + 1) / (x + 1)

Discussion

The three questions in the Try This Task! illustrate how the tasks investigated using numbers generalize to broader expressions within algebra. The first question raises issues of equivalent expressions. When written as an equation x + x = 2*x, the statement of this relationship is an identity. The second question illustrates that an equation might be valid for some but not all x values, and the third question illustrates that an equation might not be valid for any x values at all.

Typically, students initially encounter symbolic equations in the context where a variable is a place holder and the variable is 'solved for'. The expression balance highlights that an equation can be thought of as a relationship between two symbolic expressions. This method of thinking about equations helps students to make the transition to exploring the solutions of equations either graphically or numerically.

The tools provide connections from the concrete experience of balance to abstract investigations using symbols and multiple representations. These tools are only meant to aid in the transition to more sophisticated tools.

NCTM Standards and Expectations

Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope.

Recognize and generate equivalent forms for simple algebraic expressions and solve linear equations.

This lesson was developed by Gary Martin and Brian Keller.