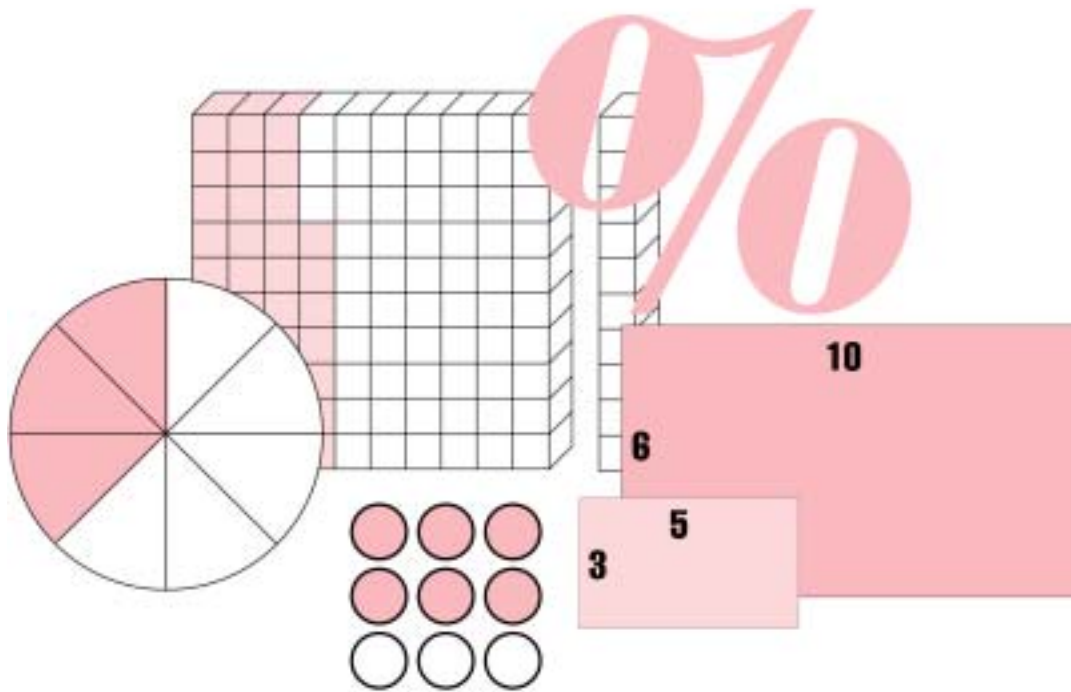


Thinking Rationally about Fractions, Decimals, and Percent



Instructional Activities for Grades 4 through 8

**Thinking Rationally about
Fractions, Decimals, and Percent:**
Instructional Activities for Grades 4 through 8



Developed by

Offices of Middle and Elementary Instructional Services
Virginia Department of Education
Richmond, Virginia

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Introduction

Thinking Rationally about Fractions, Decimals, and Percent: Instructional Activities for Grades 4 through 8 is intended to assist elementary and middle school teachers in implementing the Virginia Standards of Learning for mathematics by providing additional strategies and approaches to standards instruction in the areas of fractions, decimals, percent, and proportional thinking. These activities, organized by activity and topic, address one or more standards at each grade level and serve to introduce, reinforce, and enhance student understanding of concepts covered as part of the local curriculum. The activities may be duplicated for use in Virginia classrooms.

Teachers should reference the *Mathematics Standards of Learning Sample Scope and Sequence* document in which specific topics — e.g., fractions, decimals — are listed: this will help in presenting the necessary information in a logical, sequential, and meaningfully organized manner. Teachers should also employ the *Mathematics Standards of Learning Curriculum Framework*, which can aid lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need. This supplemental framework delineates in greater detail the minimum content that all teachers should teach and all students should learn.

Teachers are encouraged to modify and adapt these activities to meet the needs of their students. Although most of the activities are related to grade-level-specific Standards of Learning, they are not intended to constitute the total curriculum for fractions at any grade level.

This document, in addition to the *Mathematics Standards of Learning Sample Scope and Sequence* and the *Mathematics Standards of Learning Curriculum Framework*, can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at <http://www.pen.k12.va.us>.

Table of Contents

Correlation of Instructional Activities to the Virginia Mathematics Standards of Learning	1
Mathematics K–8 Standards of Learning Focusing on Fractions, Decimals, Proportion, and Percent	3
All Cracked Up	7
Animals Count	11
Area Model of Multiplication	14
Around the World	17
Building Towers.....	22
Covering the Whole Unit	25
Cover-up	27
Decimal Multiplication	30
Dividing Fractions, Using Pattern Blocks.....	32
Egg-Carton Addition.....	36
Egg-Carton Fractions	40
Four in a Row	43
Fraction Strips.....	46
Fraction-Strip Addition	49
Fraction-Strip Subtraction.....	61
Fraction Riddles	63
Gridlock	65
Measuring Cups	67
Museum Walk.....	70
Paper-Folding Multiplication.....	71
Percent Grid Patterns	75
Playground Problem.....	79
Population Density.....	83
The Scoop on Ice Cream.....	84
Snowy Egrets	88
Something’s Fishy	91
Spin To Win.....	101
The In-between Game.....	105
Virginia Population Density	109
Waste Paper	112
Which Is Closer?.....	115
Which Is More?.....	120
Who Has 100 Things?.....	123

Correlation of Instructional Activities to the Virginia Mathematics Standards of Learning for Grades 4 through 8

Activity & Page No.	Related Standards of Learning	Topic
All Cracked Up 7	4.2a, 4.3 (Geometry 4.17, 5.15, 6.14)	Area Model
Animals Count 11	6.2, 7.6, 8.3, 8.17	Proportional Thinking
Area Model of Multiplication 14	5.4, 6.6, 6.7	Area Model
Around the World 17	4.2, 4.4, 5.1, 6.1, 6.7	Connections among Fractions, Decimals, and Percent
Building Towers 22	6.4, 7.1	Comparing Fractions and Decimals
Covering the Whole Unit 25	4.2	Equivalent Fractions
Cover-up 27	4.2	Measurement Linear Model
Decimal Multiplication 30	5.1, 5.4, 6.6	Area Model
Dividing Fractions, Using Pattern Blocks 32	6.6, 7.4, 8.3	Operations with Fractions
Egg-Carton Addition 36	4.2, 4.9, 5.7, 6.6	Operations with Fractions
Egg-Carton Fractions 40	4.2, 4.3	Part-Whole Concept
Four in a Row 43	4.9, 5.7, 6.6	Operations with Fractions (Addition and Subtraction)
Fraction Strips 46	4.2, 4.3, 4.9, 6.4	Area Model
Fraction-Strip Addition 49	4.2, 4.9, 5.7, 6.6	Operations with Fractions
Fraction-Strip Subtraction 61	4.3, 4.9, 5.7, 6.6	Operations with Fractions
Fraction Riddles 63	4.2, 4.9, 5.7, 6.6	Operations with Fractions (Addition and Subtraction)
Gridlock 65	6.2, 7.6, 8.17	Proportional Thinking
Measuring Cups 67	6.6a, 6.7, 7.4, 8.3	Operations with Fractions (Division)
Museum Walk 70	6.1, 7.1	Connections among Fractions, Decimals, and Percent
Paper-Folding Multiplication 71	4.2, 4.3, 6.4	Operations with Fractions (Multiplication) Area Model
Percent Grid Patterns 75	4.4, 6.1, 7.1	Connections among Fractions, Decimals, and Percent Area Model
Playground Problem 79	6.6a, 6.8, 7.4a, 8.3	Operations with Fractions (Addition and Subtraction) Problem Solving
Population Density 83	6.2, 7.6, 8.3, 8.17	Proportional Thinking
The Scoop on Ice Cream 84	7.4, 7.6, 8.3, 8.17	Proportional Thinking
Snowy Egrets 88	6.2, 6.8, 7.6, 8.3, 8.17	Proportional Thinking
Something's Fishy 91	4.2, 4.3	Equivalent Fractions Area/Region Model
Spin To Win 101	4.2a, 4.3, 6.4	Comparing and Ordering Fractions
The In-between Game 105	4.2, 4.4, 5.1, 6.4, 7.1	Decimal Relationships
Virginia Population Density 109	6.2, 7.6, 8.3, 8.17	Proportional Thinking
Waste Paper 112	7.6, 8.3, 8.17	Proportional Thinking
Which Is Closer? 115	4.2, 4.3, 4.9, 5.7	Benchmarks for Fractions
Which Is More? 120	4.3	Comparing Fractions
Who Has 100 Things? 123	4.2, 5.2, 6.1, 7.1	Connections among Fractions, Decimals, and Percent

Mathematics K-8 Standards of Learning

Focusing on Fractions, Decimals, Proportion, and Percent

Number and Number Sense

- 1.6 The student will identify and represent the concepts of one-half and one-fourth, using appropriate materials or a drawing.

Number and Number Sense

- 2.4 The student will identify the part of a set and/or region that represents fractions for one-half, one-third, one-fourth, one-eighth, and one-tenth and write the corresponding fraction.

Number and Number Sense

- 3.5 The student will
- a) divide regions and sets to represent a fraction; and
 - b) name and write the fractions represented by a given model (area/region, length/measurement, and set). Fractions (including mixed numbers) will include halves, thirds, fourths, eighths, and tenths.
- 3.6 The student will compare the numerical value of two fractions having like and unlike denominators, using concrete or pictorial models involving areas/regions, lengths/measurements, and sets.
- 3.7 The student will read and write decimals expressed as tenths and hundredths, using concrete materials and models.

Computation and Estimation

- 3.11 The student will add and subtract with proper fractions having like denominators of 10 or less, using concrete materials and pictorial models representing areas/regions, lengths/measurements, and sets.
- 3.12 The student will add and subtract with decimals expressed as tenths, using concrete materials, pictorial representations, and paper and pencil.

Number and Number Sense

- 4.2 The student will
- a) identify, model, and compare rational numbers (fractions and mixed numbers), using concrete objects and pictures;
 - b) represent equivalent fractions; and
 - c) relate fractions to decimals, using concrete objects.
- 4.3 The student will compare the numerical value of fractions (with like and unlike denominators) having denominators of 12 or less, using concrete materials.
- 4.4 The student will
- a) read, write, represent, and identify decimals expressed through thousandths
 - b) round to the nearest whole number, tenth, and hundredth; and
 - c) compare the value of two decimals, using symbols ($<$, $>$, or $=$), concrete materials, drawings, and calculators.

Computation and Estimation

- 4.9 The student will
- add and subtract with fractions having like and unlike denominators of 12 or less, using concrete materials, pictorial representations, and paper and pencil;
 - add and subtract with decimals through thousandths, using concrete materials, pictorial representations, and paper and pencil; and
 - solve problems involving addition and subtraction with fractions having like and unlike denominators of 12 or less and with decimals expressed through thousandths, using various computational methods, including calculators, paper and pencil, mental computation, and estimation.

Number and Number Sense

- 5.1 The student will
- read, write, and identify the place values of decimals through thousandths;
 - round decimal numbers to the nearest tenth or hundredth; and
 - compare the values of two decimals through thousandths, using the symbols $>$, $<$, or $=$.
- 5.2 The student will
- recognize and name commonly used fractions (halves, fourths, fifths, eighths, and tenths) in their equivalent decimal form and vice versa; and
 - order a given set of fractions and decimals from least to greatest. Fractions will include like and unlike denominators limited to 12 or less, and mixed numbers.

Computation and Estimation

- 5.4 The student will find the sum, difference, and product of two numbers expressed as decimals through thousandths, using an appropriate method of calculation, including, paper and pencil, estimation, mental computation, and calculators.
- 5.6 The student, given a dividend expressed as a decimal through thousandths and a single-digit divisor, will find the quotient.
- 5.7 The student will add and subtract with fractions and mixed numerals, with and without regrouping, and express answers in simplest form. Problems will include like and unlike denominators limited to 12 or less.

Number and Number Sense

- 6.1 The student will identify representations of a given percent and describe orally and in writing the equivalent relationships among fractions, decimals, and percents.
- 6.2 The student will describe and compare two sets of data, using ratios, and will use appropriate notations, such as a/b , a to b , and $a:b$.
- 6.4 The student will compare and order whole numbers, fractions, and decimals, using concrete materials, drawings or pictures, and mathematical symbols.

Computation and Estimation

- 6.6 The student will
- a) solve problems that involve addition, subtraction, multiplication, and/or division with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less, and express their answers in simplest form; and
 - b) find the quotient, given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with exactly one non-zero digit.
- 6.7 The student will use estimation strategies to solve multistep practical problems involving whole numbers, decimals, and fractions (rational numbers).
- 6.8 The student will solve multistep consumer-application problems involving fractions and decimals and present data and conclusions in paragraphs, tables, or graphs. Planning a budget will be included.

Number and Number Sense

- 7.1 The student will compare, order, and determine equivalent relationships among fractions, decimals, and percents, including use of scientific notation for numbers greater than 10.
- 7.2 The student will simplify expressions that contain rational numbers (whole number, fractions, and decimals) and positive exponents, using order of operations, mental mathematics, and appropriate tools.

Computation and Estimation

- 7.4 The student will
- a) solve practical problems using rational numbers (whole numbers, fractions, decimals) and percents; and
 - b) solve consumer-application problems involving tips, discounts, sales tax, simple interest.
- 7.6 The student will use proportions to solve practical problems, including scale drawings, that contain rational numbers (whole numbers, fractions, and decimals) and percents.

Number and Number Sense

- 8.1 The student will
- a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers;
 - b) recognize, represent, compare, and order numbers expressed in scientific notation; and
 - c) compare and order decimals, fractions, percents, and numbers written in scientific notation.

Computation and Estimation

- 8.3 The student will solve practical problems involving rational numbers, percents, ratios, and proportions. Problems will be of varying complexities and will involve real-life data, such as finding a discount and discount prices and balancing a checkbook.

Patterns, Functions, and Algebra

- 8.17 The student will create and solve problems, using proportions, formulas, and functions.

All Cracked Up

Reporting Category

Number and Number Sense, (Geometry)

Overview

Students will create a set of tangrams and use them to solve a problem.

Related Standards of Learning

4.2a, 4.3 (4.17, 5.15, 6.14)

Approximate Time

45 minutes

Objectives

- Students will follow oral instructions and create a set of tangrams.
- Students will determine fractional parts of a whole, using tangrams.

Materials Needed

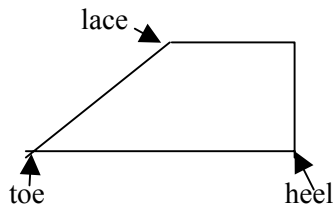
- 10 cm-by-10 cm squares of construction paper or square pieces of origami paper
- Scissors
- A copy of the handout “All Cracked Up — Area and Fractions with Tangrams” for each student
- A letter-size, three-hole-punched envelope for each student to hold tangrams for future projects

Instructional Activity

Note: Directions in italics represent a review of geometry.

1. Give each student a 10 cm-by-10 cm square piece of paper. *Talk about the attributes of a square and the ways to verify that the paper actually is a square.*
2. Have students fold the square along a diagonal. *It may be necessary to explain that a diagonal is the segment that joins two non-adjacent vertices.* Have the students cut along the creased diagonal. Model this with a square of your own (Figure 1).
3. Discuss briefly with the students the two geometric figures they have now created — two congruent isosceles right triangles — and what they know about them. *Demonstrate the concept of congruence by placing one triangle on top of the other so that it looks like only one triangle.*
4. Have the students place one triangle aside and place the other in front of them with the long side (the hypotenuse) toward their stomach, parallel to the edge of their desk. Have them fold this triangle along the altitude from the vertex angle. *It may be necessary to explain the meaning of “altitude from the vertex angle.”* Have the students crease and cut along the altitude. Model this with your own triangle (Figure 2).
5. Discuss briefly the two geometric figures they have now created — two congruent isosceles right triangles — and what they know about them. Again demonstrate the concept of congruence by placing one triangle on top of the other so that it looks like only one triangle. *Depending on the grade level, ask students if these right triangles are congruent to the one they put aside. This will provide the opportunity to discuss the difference between congruence and similarity. Demonstrate similarity by showing that the angles are congruent.* Have the students label these triangles “1” and “2.”
6. Have the student place the remaining large right triangle in front of them with the long side (the hypotenuse) toward their stomach. Have them fold the vertex of the right angle down to the midpoint of the opposite side. Model this with your own triangle. Show them how it should look — a small triangle folded over a trapezoid. Cut along the crease. Label the new triangle “3” (Figure 3).
7. Discuss briefly the two geometric figures they have now created — an isosceles right triangle and an isosceles trapezoid. *If some students do not know what a trapezoid is, introduce it.*
8. Have the students place the trapezoid in front of them with the longer side toward their stomach. Have them fold the trapezoid along the height of the trapezoid that connects the midpoints of the two bases. Model this with your own trapezoid. Cut along the crease. (Figure 4).

9. Discuss with the students the two geometric figures they have now created — two congruent trapezoids. *This is a good time to discuss the similarities in and differences between the two trapezoids they have seen thus far.*
10. To assist students in following the next steps, place the following diagram of a “shoe” on the board or overhead.



Say to students: “These are what we are now going to call shoes. You have two of them. Place one in front of you and put the other aside. Take the toe and fold it back to the heel.” Model with your own shoe. When the fold is complete, it should look like a triangle on top of a square. Cut along the crease. Label the triangle “4” and the square “5.” *Now is a good time to continue the discussion of geometric figures and what figures they now have — an isosceles right triangle similar to the others and a square. Ask how they can verify that they have a square (Figure 5).*

11. Have the students take the second shoe and place it in front of them, oriented like the model on the board or overhead. Have them take the heel and fold it to the lace. Model with your second shoe. *It should look like a triangle folded over a parallelogram. Cut along the crease. Label the small triangle “6” and the parallelogram “7” (Figure 6).*
12. Finish the discussion of geometric figures. *The students should now have a small isosceles right triangle and a parallelogram. Is the triangle congruent or similar to the others? Talk about the properties of a parallelogram.*
13. Have the students make sure that they each have seven labeled pieces. Then have them have put the seven pieces back together into the large square with which they started.
14. At the end of this activity, give each student an envelope with holes punched in it for a three-ring binder, and have them store their pieces for future work.

Figure 1.

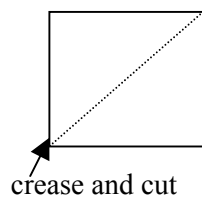


Figure 2.

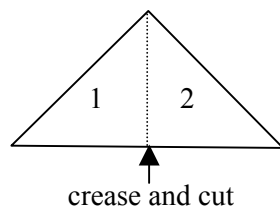


Figure 3.

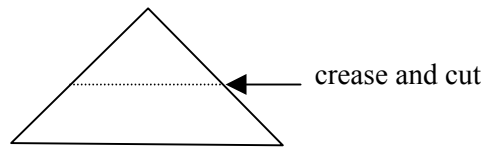


Figure 4.

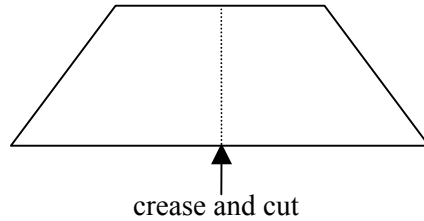


Figure 5.

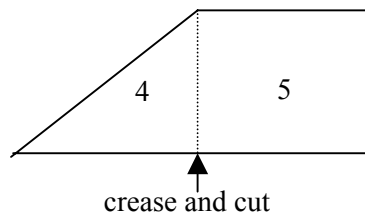
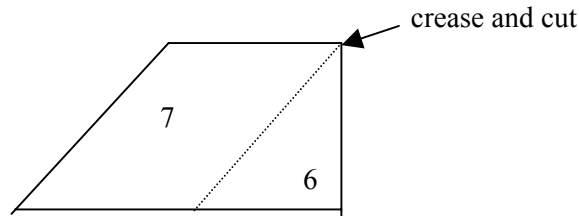


Figure 6.



Classroom Assessment

Watch carefully as students follow your instructions. Answer all questions that they may ask. Circulate to be sure that all students are understanding and folding correctly. As students try to reassemble the square, circulate and talk with them about the strategies they are using. Distribute a copy of the handout "All Cracked Up — Area and Fractions with Tangrams" to each student. While the students are completing the handout, circulate and again discuss the strategies they are using to determine the fractional part of each piece.

Follow-up/Extensions

As indicated in the directions in italics, this is a good activity for a review of geometry.

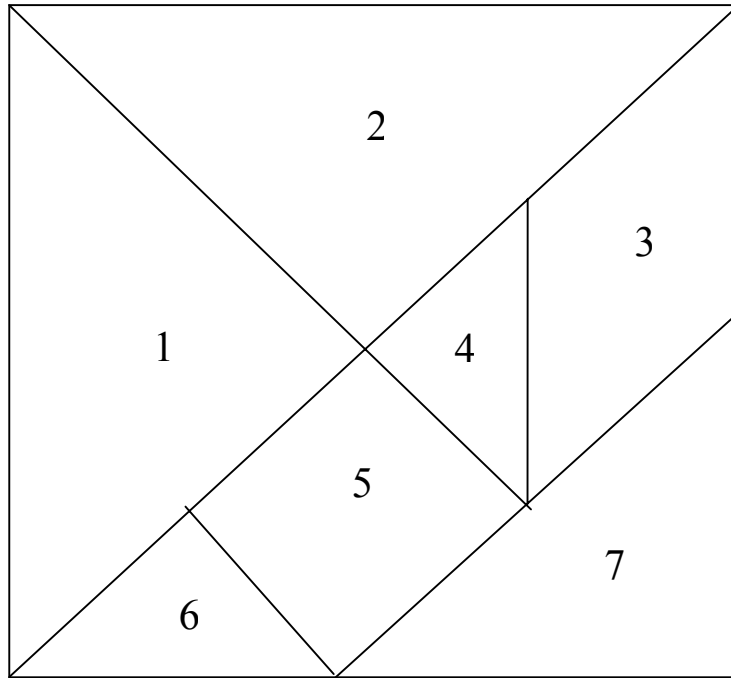
Curriculum Connections

Literature: Tompert, Ann. *Grandfather Tang's Story*. New York: Crown, 1990.

All Cracked Up — Area and Fractions with Tangrams

Steps

1. Identify each tangram piece by the name of its shape.
2. Which pieces are the same size? How do you know?
3. If the area of the whole piece is 1 (one) unit, find the area of each numbered piece.



Piece #	Name	Area
1	_____	_____
2	_____	_____
3	_____	_____
4	_____	_____
5	_____	_____
6	_____	_____
7	_____	_____

Animals Count

Reporting Categories

Number and Number Sense; Computation and Estimation; Patterns, Functions, and Algebra

Overview

Students will simulate a method employed by biologists to estimate animal populations, using beans to represent the animals. The method, called “capture-tag-recapture,” uses red and white beans to represent tagged and untagged populations of deer. During the simulation, proportions are developed for use in determining the total number of the entire population.

Related Standards of Learning

6.2, 7.6, 8.3, 8.17

Approximate Time

60 minutes

Objective

Students will use previously learned techniques for reasoning about ratios and proportions to estimate the size of a population when they cannot count the entire population.

Prerequisite Understandings/Knowledge/Skills

- Understand how to create proportions
- Understand how to solve proportions
- Understand the meaning of ratios
- Understand how to perform operations with fractions

Materials Needed

- Small jar
- Large jar or other container with lid for each group of students
- Enough white beans to fill the jar of each group
- 100 red (kidney) beans for each group
- Paper cups or scoops for scooping beans
- A copy of the handout “Animals-Count Recording Sheet” for each group

Note: Two types (colors) of GoldfishTM crackers or other equal-sized objects can be substituted for beans.

Instructional Activity

1. *Initiating Activity:* Discuss with the class: “How do wildlife officials determine how many sea otters there are in the United States?” “How do environmentalists decide whether an animal or plant should be on the endangered species list?” Answers will vary. Discuss scientific methodology for performing accurate counts of wildlife.
2. Demonstrate the experiment, using a small jar that is three-fourths full of white beans. Remove 10 beans and replace them with red beans. Cover the jar, shake, and remove a sample handful of beans.
3. Ask students to count the number of red beans and white beans in the sample. How can they set up a meaningful ratio using these numbers?

$$\text{Ratio} = \frac{\text{number of red beans in sample}}{\text{total number of beans in sample}}$$

Be sure the students recognize that at this point they know a) the number of marked (red) beans in the whole population and b) the number of marked (red) beans in the sample.

4. Ask the students to use what they know about making comparisons with ratios to estimate the total number of beans in the jar. Record their proportion and their estimate. Students should be able to record a proportion that shows the total number of red beans drawn in four trial samples related to the total

number of all beans drawn in the four samples. This ratio is equal to the ratio of the total number of red beans (100) to the total population, which is the number being sought, x .

$$\text{Proportion: } = \frac{\text{total number of red beans (100)}}{\text{total population (} x \text{)}}$$

5. Have a group count the number of beans in the jar to see how close the estimate was to the actual number of beans.
6. Discuss with students the methodology shown by the experiment, which models what really happens when the capture-tag-recapture method is used to estimate the number of deer in a large area. Some factors that must be considered are tagging deer from several places in the area under study, taking a sufficient sample for tagging, allowing the tagged animals time to mix thoroughly with the population, and taking the final samples from several places in the area.
7. *Simulation Activity:* Provide each group of three or four students with a jar with a lid, a large number of white beans, and an “Animals-Count Recording Sheet.” Model the first trial of the deer-population experiment with the students, making sure that they know how to record their results.
8. Ask the groups of students, “How are you mixing the beans?” “How are you keeping track of what you know?” “How might the number of samples you take affect your estimate?”
9. Have each group remove 100 white beans from the jar and set them aside.
10. Have them place 100 red beans representing the tagged deer into the jar to replace the white beans removed.
11. Have them shake the jar to mix the beans and then scoop out a cupful of beans without looking at them.
12. Have them record on the recording sheet the number of red beans and the total number of beans in the sample.
13. Have them repeat this scoop-and-count procedure three more times. In each case, have them record on the recording sheet the number of red beans and the total number of beans in the sample.
14. *Closing Activity:* Have the groups study the data they collected and use the data to estimate the number of beans in the jar. Have each group report, explaining how they made their estimate and showing any calculations they made. The proportions should use the reasoning about the tagged deer and the total population as shown in the sample (see Initiating Activity, steps #3 and 4).

Classroom Assessment

As student groups are working, circulate in the room to make sure they are accurately recording all needed information and to assess their understanding of ratios. As groups report, make sure their proportions make sense mathematically. Students should be able to articulate their answers and how they relate to the proportionality of the capture-tag-recapture method.

Follow-up/Extensions

Additional problems using this method could be assigned for students struggling with forming their proportions. For instance, similar real-life problems might involve fish in a lake or beavers in a wooded area.

Curriculum Connections

Science: Population disturbances and factors that increase or decrease population size

Social Studies: Census taking, population studies



Animals-Count Recording Sheet

To simulate a method employed by biologists to estimate animal populations, we will perform an experiment using beans to represent the animals. The method is called “capture-tag-recapture.” In this simulation, the red beans represent the deer that are tagged by the conservation officer and the white beans represent the deer that are not tagged. All the beans in the jar represent the total population of deer in a given region, which is the unknown quantity we seek to estimate.

Part One:

1. Fill a jar with a large number of white beans.
2. Remove 100 white beans from the jar and set them aside.
3. Place 100 red beans into the jar to replace those white beans removed. These red beans represent the tagged deer.
4. Put a lid on the jar and shake it to mix the beans.
5. Without looking at the beans, scoop out a sample cupful of beans.
6. Count and record in the chart below the number of red beans and the total number of beans in your sample.
7. Return the sample to the jar.
8. Shake the jar again to mix the beans.
9. Repeat the scoop-and-count procedure three more times. For each trial, record the number of red beans and total number of beans in the sample.

Study the data you collected. Use the data to estimate the total number of beans (the total deer population) in your jar. How did you decide? Explain how you made your estimate, show your calculations, and explain your strategy.

	Sample 1	Sample 2	Sample 3	Sample 4
Number of Red Beans in Sample				
Total Number of Beans in Sample				
Ratio				

Area Model of Multiplication

Reporting Categories

Number and Number Sense, Computation and Estimation

Overview

Students will relate the area of a rectangle to the multiplication of fractions or decimals.

Related Standards of Learning

5.4, 6.6, 6.7

Approximate Time

45 minutes

Objectives

- Students will gain experience in visualizing the meaning of multiplication of fractions.
- Students will connect the meaning of multiplication of fractions to the multiplication of whole numbers.

Prerequisite Understandings/Knowledge/Skills

- Understand how to compare the numerical values of two fractions having like and unlike denominators, using pictorial representations
- Understand multiplication of whole numbers
- Understand and use the terms *fraction*, *whole number*, *numerator*, and *denominator*
- Understand how to divide regions and sets to represent a fraction
- Understand how to find the area of a rectangle

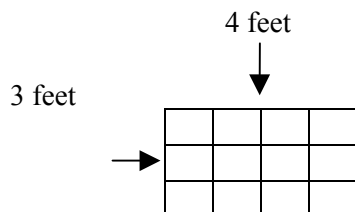
Materials Needed

- Plain copy paper
- Pencils
- Colored pencils
- Rulers (optional)
- Blank transparencies
- Overhead and regular markers
- Chart paper
- Sticky notes

Note: It is important to script this activity carefully and thoroughly for the students. The value of this activity is dependent upon clearly relating the divisions of paper to the operation of multiplication. Each step of the operation should be fully discussed.

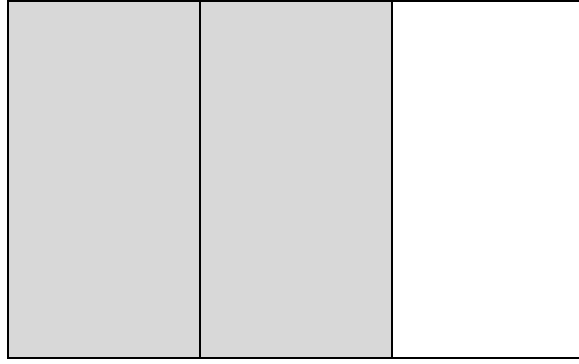
Instructional Activity

1. *Initiating Activity:* Lead the students in the following dialogue and steps: “We are using an area model for fractions. To understand this better, we should look at whole number multiplication as it relates to the area of a rectangle. How do we find the area of a rectangular piece of tile that is 3 feet wide by 4 feet long?” Demonstrate this on the board or overhead, as follows:

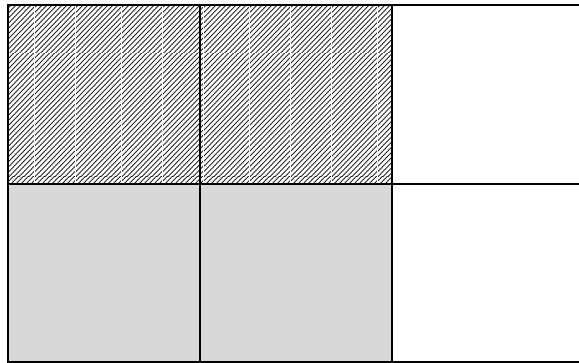


Notice that the area contains 12 squares or 12 square feet. We multiplied 3 feet by 4 feet to get the total of 12 square feet.

2. “Consider the problem $\frac{1}{2}$ of $\frac{2}{3}$. We can think of this as finding the area of a rectangle with a height of $\frac{1}{2}$ and a width of $\frac{2}{3}$.”
3. “Draw a rectangle. Separate it with two vertical lines into three equal-sized sections, and shade or color two adjacent sections.”



4. “The shaded portion represents which fraction?” ($\frac{2}{3}$).
5. “We can represent the first fraction ($\frac{1}{2}$) by separating the original rectangle with a horizontal line into two equal-sized sections. In this case, however, we will use stripes to shade only that part of the picture that shows a fraction of the original fraction — that is $\frac{1}{2}$ of the $\frac{2}{3}$.”



6. “What is the width of the rectangle that is striped?” ($\frac{2}{3}$ of a unit)
7. “What is the height of that rectangle?” ($\frac{1}{2}$ of a unit)
8. “What is the area of that rectangle?” “Count the equal portions of the original rectangle that are striped.” (2 out of 6, or $\frac{2}{6}$, or $\frac{1}{3}$)
9. “How does this answer relate to the original problem $\frac{1}{2} \times \frac{2}{3}$?” (It is equivalent.)
10. Discuss the equivalence thoroughly. If necessary, model additional problems.

11. *Closing Activity:* Ask the participants to work in groups to draw the following problems on chart paper. (Write the problems on the overhead for groups to copy). Post students' work on the board, and allow everyone to walk around and view the work of other groups. Use sticky notes or markers to point out any discrepancies or disagreements. Hold a class discussion after everyone has had the opportunity to work all the problems.

$$\frac{1}{3} \times \frac{3}{4}$$

$$\frac{1}{2} \times \frac{1}{4}$$

$$\frac{1}{4} \times \frac{3}{5}$$

$$\frac{1}{2} \times \frac{5}{6}$$

Classroom Assessment

Circulate around the classroom, observing students as they complete each part of the activity. Watch for understanding of each concept, pose questions about the activity, and encourage student explanations and questions. Note any misunderstandings and correct those immediately by engaging students in additional drawings and pictorial models.

Follow-up/Extensions

Use activities from *Fabulous Fractions* (AIMS) that provide opportunities for students to engage in area models for multiplication. A specific extension activity that includes modeling multiplication of fractions is "Fair Square and Cross Products." Students should be encouraged to create "Poster Proofs" that show their understanding of area models for multiplication.

Curriculum Connections

Social Studies: Quilting in the Appalachian Region

Science: Cooking

Literature: Math journals

Giganti, Paul. *Each Orange Has 8 Slices*.

Seymour, Dale. *Different Shapes, Equal Pieces: Fractions and Area*.

Scinto, Joyce. *Math Stories: Fractions, Decimals, Percents*.

Adler, David. *Fraction Fun*.

Around the World

Reporting Categories Overview

Number and Number Sense, Computation and Estimation
Students will determine what percent of the time their right index finger will come to rest on land or water when catching a tossed globe. Students will develop percent number sense by first estimating and then actually catching the globe and recording where the right index finger lands each time.

Related Standards of Learning 4.2, 4.4, 5.1, 6.1, 6.7

Approximate Time 45 minutes

Objectives

- Students will develop decimal number sense and make connections among different representations — fractions, decimals, and percents — for the same amount.
- Students will use estimation to relate percents to land and water areas around the world.

Prerequisite Understandings/Knowledge/Skills

- Understand that decimals are another way of representing fractions
- Understand that decimals are written as an extension of the place-value system and that each place to the right of the decimal gets ten times smaller than the previous place

Materials Needed

- One inflatable plastic globe for each of three groups of students
- A copy of the handout “Around-the-World Recording Sheet 1” for each student

Instructional Activity

1. *Initiating Activity:* Have students predict the percent of land versus water found on the globe. Record a few of these predictions on the board for future reference.
2. Explain to students that they will be collecting data to verify their predictions. Organize students into three groups, and give each group a plastic globe and a set of recording sheets.
3. Prior to beginning the tosses, have the students record their individual predictions on their recording sheet. The prediction is a guess as to how many times out of 100 catches the student’s right index finger will touch land and how many times it will touch water.
4. Explain the data collecting process to the students. Have each group form a circle and toss the globe back and forth among the members for a total of 100 tosses. For each toss, have one student in each group record the data by putting a tally mark in the appropriate space on his/her recording sheet.
5. After the 100 tosses and all data collecting are completed, have the students in each group transcribe the total of the tally marks onto their own recording sheets. Then show them how to convert the data into the correct percents. Model the conversion with the following example: 72 tosses came to rest on water; 28 tosses came to rest on land. Record the data as 72 out of 100 equals $\frac{72}{100}$, .72, and 72%; and 28 out of 100 equals $\frac{28}{100}$, .28, and 28%. Students may represent the fraction and decimal by using place value materials or 100 grids.

6. Have each group share their results, and display all the data in a class chart.
7. *Closing Activity:* Using the results on the class chart, have students compare the results to the original predictions. If large differences exist between the predictions and the actual results, discuss why the differences exist.

Classroom Assessment

During the activity, observe students as you walk around the room and check for understanding. At the end of the activity, students may respond to the following prompts in their math journals, “How are fractions, decimals, and percents related?” “What connections are there among fractions, decimals, and percents?”

Follow-up/Extensions

This activity can be extended so that students make predictions about what percent of land is occupied by the individual continents and what percent is occupied by individual oceans. Refer to chart “Area of Land and Water around the World,” and have the students use the “Around-the-World Recording Sheet 2.” The class could be divided into two groups, each of which does 50 tosses, and then the results can be added together (averaged).

Curriculum Connections

Science: Earth and Space Science topics

Social Studies: Map reading

Around-the-World Recording Sheet I

1. Estimate the number of times out of 100 catches that your right index finger will come to rest on water.
2. Estimate the number of times out of 100 catches that your right index finger will come to rest on land.
3. Toss and catch the globe 100 times.
4. Keep a tally or actual count and make a record of where your right index finger comes to rest each time.
5. Figure the fraction, decimal, and percent representations of the number of times your finger came to rest on land; then figure the same representations for water.

	Estimate	Actual	Fraction	Decimal	Percent
Land		Tally marks: _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ Total:			
Water		Tally marks: _____ _____ _____ _____ _____ _____ _____ _____ _____ Total:			
Total	100	100	$\frac{100}{100}$	1.00	100%

Around-the-World Recording Sheet 2

1. Estimate the number of times out of 100 catches that your right index finger will come to rest on each of the continents shown below.
2. Toss and catch the globe 100 times.
3. Keep a tally or actual count and make a record of where your right index finger comes to rest each time.
4. Figure the percent representations of the number of times your finger came to rest on each continent.
5. Explain any discrepancies between your estimated and actual counts.

	Estimate	Actual	Percent
Asia			
Africa			
Antarctica			
Australia			
Europe			
North America			
South America			
All Land			
All Water			

Area of Land and Water around the World

Total Square Miles on Earth = 195,331,609

Continent	Square Miles	Percent of Earth
Asia	16,957,000	9%
Africa	11,704,000	6%
Antarctica	5,100,000	3%
Australia	2,967,909	2%
Europe	4,063,000	2%
North America	9,416,000	5%
South America	6,888,000	4%
Land Total	57,095,909	31%

Water	Square Miles	Percent of Earth
Pacific Ocean	64,186,000	32%
Atlantic Ocean	33,420,000	17%
Indian Ocean	28,350,000	14%
Arctic Ocean	5,105,700	2%
Seas, Gulfs, and Bays	7,174,000	4%
Water Total	138,235,700	69%

Building Towers

Reporting Category Overview

Number and Number Sense

Students will convert fractions to decimals, using calculators, in the context of a game with the goal of building as high a decimal/fraction tower as possible.

Related Standards of Learning

6.4, 7.1

Approximate Time

30 minutes

Objective

Students will develop an understanding of how to compare two fractions and their equivalent decimal representations.

Prerequisite Understandings/Knowledge/Skills

- Understand that a fraction is a way of representing part of a unit whole
- Understand that models can be used to judge the sizes of fractions and decimals
- Understand benchmarks as an aid in comparing the sizes of fractions and decimals

Materials Needed

- Transparency of “Tower”
- Pencil and paper
- Calculators
- A copy of the handout “Building Towers” for each student

Instructional Activity

1. *Initiating Activity*: Demonstrate to students on the overhead how to play the “Building Towers” game. Model the activity by playing the game with one of the students.
2. Rules of the game:
 - a. The first player picks two different whole numbers from 0 to 99, say 2 and 3. The first player then divides the smaller number by the larger number, using the calculator ($2 \div 3 = .667$). He/she then records the fraction and its decimal equivalent on the top story of the tower.

$$\frac{2}{3} \approx .667$$

- b. The second player then builds the next-to-the-top story of the tower. The second story decimal needs to be greater than the decimal in the top story, but less than one. For example the second player picks 5 and 6, which makes the fraction $\frac{5}{6}$. Calculating the decimal equivalent produces .833.

These numbers are recorded in the second story.

$$\frac{2}{3} \approx .667$$
$$\frac{5}{6} \approx .833$$

- c. The players continue taking turns and adding stories until one player cannot find a decimal that fits between the decimal of the last story and 1.
3. Pass out the handouts and calculators to the students then have them play the game with a partner.
4. *Closing Activity:* After all students have had an opportunity to play the game a few times, ask those with the tallest towers to share the strategies they used to win. Encourage the students to share different strategies to find what seems to be an efficient way to win. Record these on a chart for future play-offs.

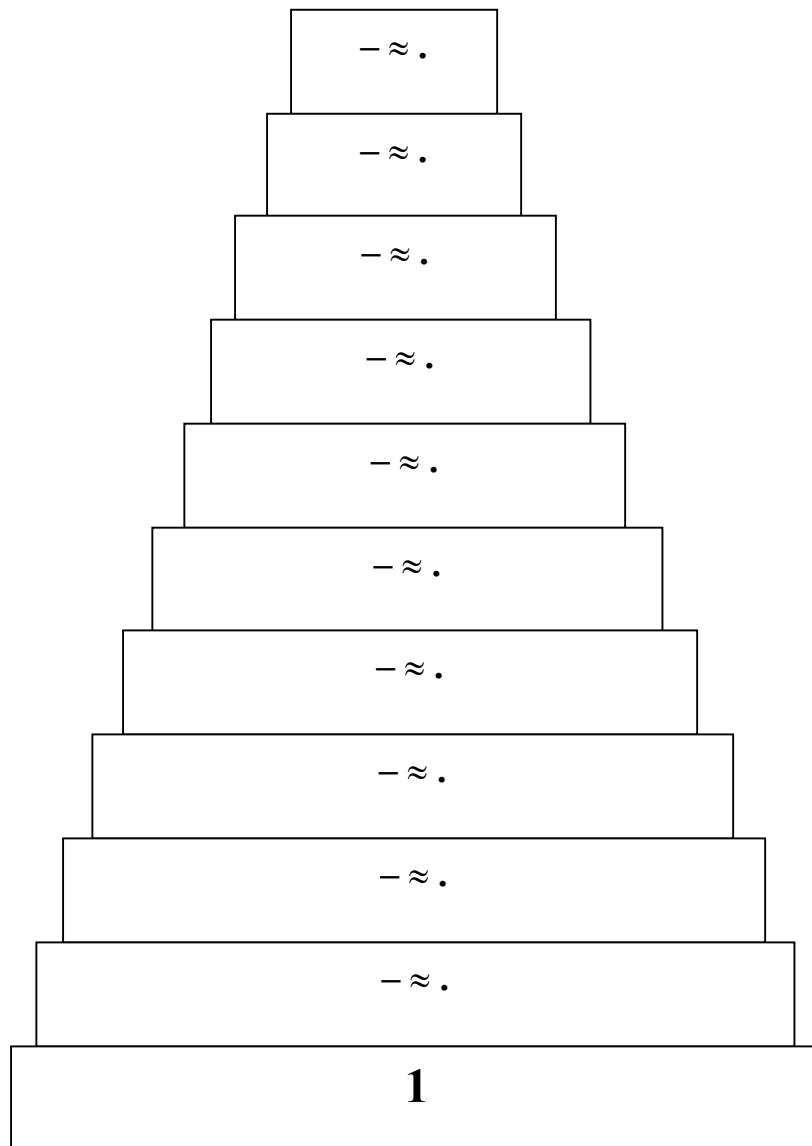
Classroom Assessment

During the activity, observe students as you walk around the room and check for understanding. At the end of the activity, students may record individual observations in their math journals. Students may record the rules of the game and take a copy of the handout to practice with a partner at home. The next day students may share the strategies the partner at home used.

Follow-up/Extensions

Extend the level of difficulty of the game by using a combination of double and single digits in creating the fractions. Have the students share if any new strategies are developed.

Building Towers



Covering the Whole Unit

Reporting Category

Number and Number Sense

Overview

Students will manipulate parts of a whole and make a whole from fractional parts. This activity is an extension of “Something’s Fishy.”

Related Standards of Learning

4.2

Approximate Time

45 minutes

Objectives

- Students will visualize equivalent fractions with fraction pieces.
- Students will make a whole from parts that require regrouping.

Prerequisite Understandings/Knowledge/Skills

- Understand the following terms: *fraction*, *whole*, *unit*, *half*, *fourth*, and *eighth*
- Identify and represent equivalent fractions, using concrete objects

Materials Needed

- Fraction strips
- Fraction dice
- One game board for each player

Instructional Activity

1. Each player will need his/her own envelope or plastic storage bag of fraction strips (perhaps created previously during other activities). Make sure the students have written their name on the back of each of their fraction strips.
2. Working in pairs or small groups of three-to-four players, each player empties all his/her fraction strips into a common pile in the center of the table. The players take turns drawing fraction strips until all the strips have been drawn.
3. The game board represents the unit 1, or one whole. The object of the activity is to be the first player to cover the whole with no overlaps.
4. The first player rolls the fraction dice and puts a fraction strip of that amount on his/her game board.
5. The second player does the same. Play continues, making equal substitutions if necessary. For example, a player with a $\frac{1}{4}$ and a $\frac{1}{2}$ already on his/her game board rolls a $\frac{1}{4}$. He can use a $\frac{1}{4}$ strip or two $\frac{1}{8}$ strips to make this move.
6. The winner is the first player to exactly cover his/her whole game board.
7. When play is complete, have the students collect all of their own fraction strips and return them to their storage bag.

Covering the Whole Unit — Fraction Strips

Cut out the following fractions strips and store them in a plastic storage bag. Tag board works best for making fraction strips.

1

$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
---------------	---------------	---------------

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$
---------------	---------------	---------------	---------------

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
---------------	---------------	---------------	---------------	----------------	----------------	----------------	----------------

$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
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Cover-up

Reporting Category Overview

Number and Number Sense

Students will use a measurement/linear model to find and record equivalent fractions within the context of a game. This activity is an extension of the activity “Something’s Fishy.”

Related Standards of Learning

4.2

Approximate Time

45 minutes

Objective

Students will explore relationships between equivalent fractions, using concrete materials.

Prerequisite Understandings/Knowledge/Skills

- Understand that equivalent fractions are ways of describing the same amount, using different-sized fractional parts
- Understand that fraction strips are an area/region fraction model that can clarify the concept of equivalent fractions

Materials Needed

- A set of fraction strips for each player
- One whole fraction piece per player, to be used as a game board.
- A copy of the handout “Cover-up Spinner” for each student
- Paper clips and pencils
- Cuisenaire rods (a collection of rectangular rods, each of a different color and size)

Instructional Activity

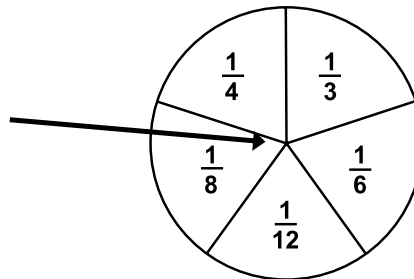
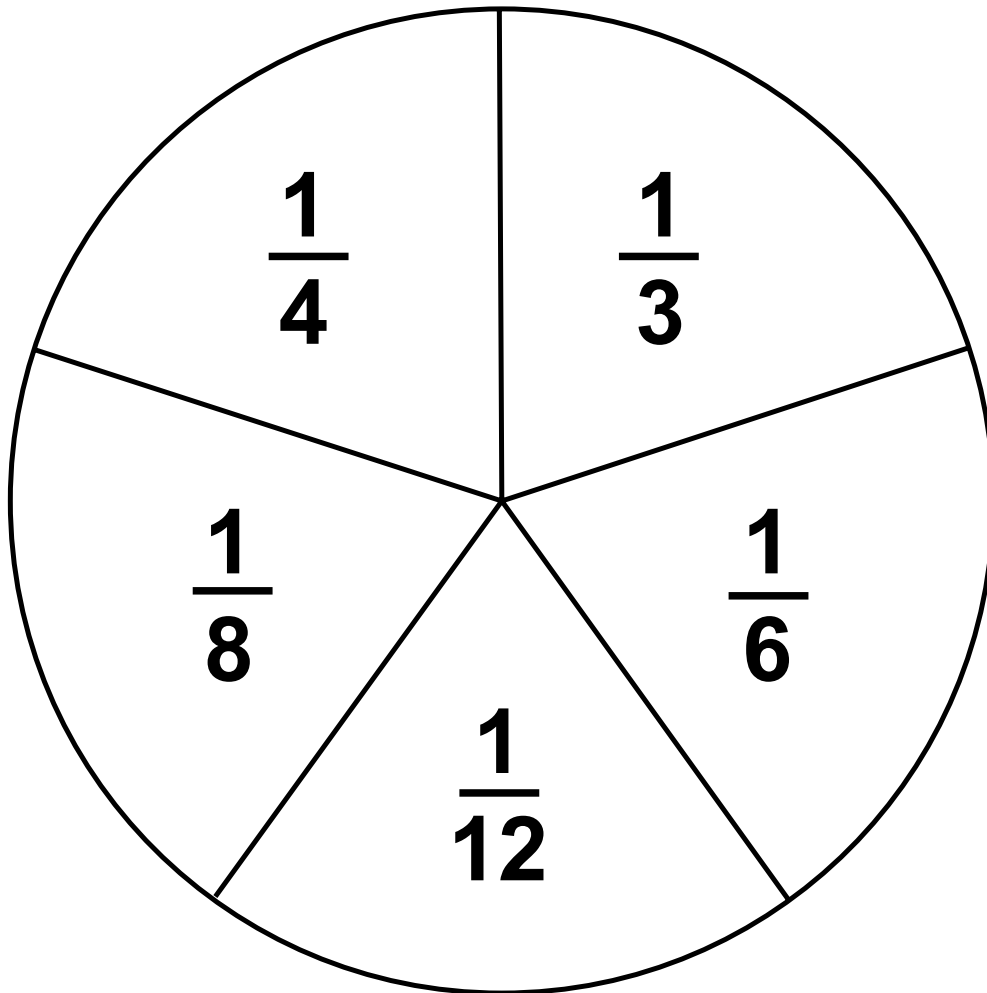
1. Ask each student to find the whole, the halves, the thirds, the fourths, the sixths, the eighths, and the twelfths from the set of cuisenaire rods.
2. Explain to the class that pairs of students will use their own sets of fraction strips to play the game “Cover-up.” Each player will start with the “whole” fraction piece as his/her individual game board. The object of the game is to be the first to cover the game board entirely with the other fraction strips. Overlapping of fraction strips is not permitted.
3. Demonstrate how to play the game. For each turn, a player spins the spinner, and the fraction shown indicates what fraction piece to place on the game board. Play continues until one person has completely covered his/her game board. When a player finds it possible to exchange multiple, same-unit fraction pieces from the game board for a *fewer* number of pieces representing an equivalent fraction, this *must* be done. For example, if a player has two $\frac{1}{8}$ pieces on his/her board, these *must* be exchanged for one $\frac{1}{4}$ piece. Remember that all replacement pieces must be the same unit fraction — e.g., four $\frac{1}{8}$ pieces may not be replaced with three $\frac{1}{12}$ pieces and one $\frac{1}{4}$ piece.
4. When the game nears the end, a player must spin the exact fraction needed in order to win. If a participant needs only a small piece such as $\frac{1}{12}$ in order to win, it is not permitted to use $\frac{1}{4}$.

5. After all participants have played one preliminary round of the game, have them play another round and record the fractional equivalents they encounter.
6. Discuss the mathematical concepts and procedures modeled in this game. Ask the students to describe how they created pairs of equivalent fractions, as well as any strategies they found useful while playing the game.

Follow-up/Extensions

Use two “wholes” instead of one to increase the level of difficulty. This game may also be used to explore the symbolism of fractions with numerators other than one and the process of comparing, ordering, adding, and subtracting fractions with like and unlike denominators.

Cover-up Spinner



Decimal Multiplication

Reporting Categories

Number and Number Sense, Computation and Estimation

Overview

Students will use base-ten materials to model the multiplication of decimals.

Related Standards of Learning

5.1, 5.4, 6.6

Approximate Time

Two 45-minute activities

Objective

Students will understand the multiplication of decimals by using a base-ten model.

Prerequisite Understandings/Knowledge/Skills

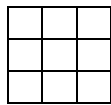
- Understand that decimals are another way of representing fractions
- Understand that decimals are written as an extension of the place-value system
- Understand that base-ten materials can be used to express decimal representations

Materials Needed

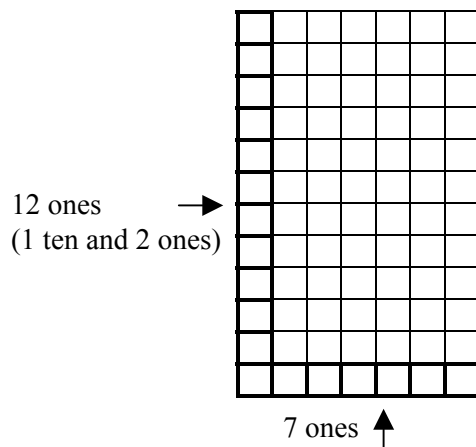
- Base-ten pieces or cardstock copies
- Overhead base-ten pieces
- Plain copy paper

Instructional Activity

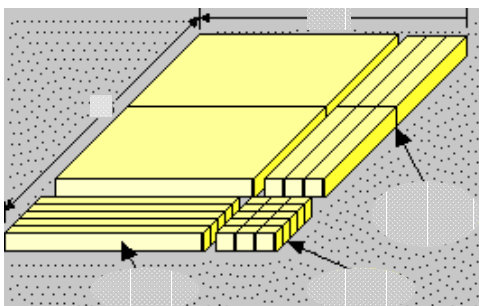
1. *Initiating Activity:* Begin the activity by explaining to students that they will be using base-ten materials to learn about multiplication of decimals. Pass out the base-ten materials and blank sheets of paper. Have students construct a place-value mat by folding the paper into three sections, labeling the sections with hundreds, tens, and ones. In order to review the use of the base-ten materials, have the students model a three-digit number, e.g., 284, by placing two flats in the hundreds place, eight rods in the tens place, and four squares in the ones place.
2. Review multiplication of whole numbers, using place-value pieces. Have students model such multiplication, using an array and whole numbers to show the multiplication of 3 ones by 3 ones.



3. Review multiplication of one-digit numbers by two-digit numbers, e.g., 7×12 . Have the students place the pieces representing seven ones on the bottom of the rectangular array and the pieces representing twelve ones (1 ten and 2 ones) on the left side of the array. Then have them complete the rectangle created with the largest pieces (rods) first and then the units needed.



4. Now students are ready to begin using the base-ten pieces to represent decimals. Ask students to suggest ways the base-ten pieces might be used to represent certain decimal numbers, for example, 1.23. Have the students turn over their place-value charts from the initiating activity and relabel the three columns with ones, tenths, and hundredths. Model how the flat now represents the ones, the rods now represent the tenths, and the units now represent the hundredths. They may also add a “decimal point” on the line between the ones and tenths columns. Have students practice representing other numbers on their decimal place-value chart before moving on to multiplication.
5. Model multiplying with decimals such as in the sample problem below. Have students create a rectangle by using 1.3 (1 flat and 3 rods) as the dimension across the top of the rectangular array and 2.5 (two flats and 5 rods) as the dimension along the side of the array.



6. Represent the multiplication in the following way:

$$1 \times 2 = 2 \text{ (the two whole flats)}$$

$$1 \times 0.5 = 0.5 \text{ (5 rods)}$$

$$0.3 \times 2 = 0.6 \text{ (6 rods)}$$

$$0.3 \times 0.5 = 0.15 \text{ (15 unit pieces)}$$

After combining pieces (exchanging the 11 rods for 1 flat and 1 rod, and the 15 unit pieces for 1 rod and 5 unit pieces) the total is 3 flats, 2 rods and 5 unit pieces, or 3.25.
7. Review the distributive property of multiplication with the students to relate this property to the previous problem. Have students continue to model the process with the following other problems. Remind the students to build rectangles, using their base-ten pieces.

$$1.4 \times 1.3$$

$$0.6 \times 1.2$$

$$0.3 \times 0.3$$

$$1.2 \times 2.5$$

$$0.4 \times 1.1$$
8. Call on individual students to model the problems for the class.
9. Ask each student to use the base-ten pieces to create a multiplication problem individually. Have each student build his/her problem and record the numerical expression and representation of the problem, as well as the answer. Have students write their problems without the answers on sticky notes and switch notes with other students so that each student has a new problem to solve. Once the problems are solved, have students compare answers with the creators of the problems.
10. *Closing Activity:* Ask students to write about the process of multiplying decimals by using the problem (1.2×2.5) and the base-ten pieces. Have students record their explanations in their journal.

Classroom Assessment

Check on individual progress by evaluating the problems on the sticky notes and the journal entries explaining the process.

Follow-up/Extensions

Continue to use the base-ten pieces in additional activities to model multiplying decimals.

Dividing Fractions, Using Pattern Blocks

Reporting Category

Computation and Estimation

Overview

Students will use pattern blocks to represent the whole and then determine a fractional amount of the whole.

Related Standards of Learning

6.6, 7.4, 8.3

Approximate Time

90 minutes (can be split into two 45-minute activities after step #7)

Objective

Students will be able to divide a whole number by a fraction.

Prerequisite Understandings/Knowledge/Skills

- Understand that a fraction is a way of representing part of a unit whole
- Understand that the denominator tells the number of equal-sized parts in a whole
- Understand that the numerator tells the number of equal-sized parts being considered
- Understand how the pattern blocks relate to each other
- Understand that division is used to describe how many of a given divisor there are in a given dividend

Note: If students do not have the prerequisite understandings, knowledge, and/or skills, refer back to previous grade-level Standards of Learning and lessons for ideas to reteach. The language of numerator and denominator is used in sixth grade.

Materials Needed

- Pattern blocks
- Graph/grid paper
- Colored pencils (optional)

Note: For easier management, put each pattern block set in a plastic storage bag for each student.

Instructional Activity

1. *Initiating Activity:* Discuss with the class: “What is division of whole numbers?” “What does 6 divided by 2 mean?” “What does 12 divided by 2 mean?” “What does 12 divided by 6 mean?” In asking these questions, you are trying to encourage student understanding that division describes how many of a given divisor there are in a given dividend. Have students represent (sketch) 6 divided by 3 and make a story problem for 6 divided by 3. Have volunteers read their story problems.
2. Have students work in groups to model, using pattern blocks, the solution to the following problem: “Duncan has four pounds of candy and decides to use it all to make $\frac{1}{3}$ -pound bags to give away. How many bags can he make with his four pounds of candy?”
 - a. Students may solve this problem by using four hexagons to model four pounds of candy. Using a rhombus to represent $\frac{1}{3}$ of a pound of candy, they will see that there are three rhombi in a hexagon and, therefore, twelve rhombi in four hexagons. Hence, one can make twelve $\frac{1}{3}$ -pound bags of candy out of four pounds of candy.
 - b. Another way students may solve this problem is to use graph paper to represent the problem. Since the problem uses division by thirds, discuss with the students why three equal-sized parts

should be used to represent each pound of candy. Colored pencils may be used, if desired. Students should also write the problem as in the following model:

one pound of candy				
one pound of candy				
one pound of candy				
one pound of candy				

$$4 \div \frac{1}{3} =$$

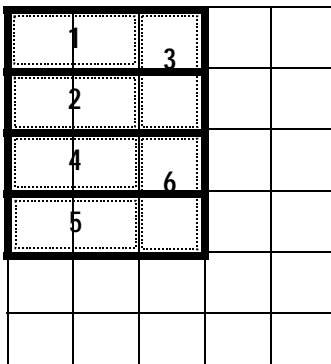
$$4 \cdot 3 = 12$$

3. Have small groups of students use pattern blocks to model the following problem and then represent it on graph paper: “Susan had three blocks of candy. She wants to divide each block into $\frac{1}{6}$ -size pieces of candy. How many pieces of candy will she be able to make?” Students may solve this problem by using three hexagonal blocks to represent the three blocks of candy. They may then use a triangular block to represent $\frac{1}{6}$ of a block and find the solution. (18 pieces of candy)
4. Have small groups of students use pattern blocks to model the following problem and then represent it on graph paper: “The Virginia Housing Company wants to divide five acres of land into $\frac{1}{2}$ -acre lots. How many lots will there be?” Students may solve this problem by using five hexagonal blocks to represent the five acres of land. They may then use a trapezoidal block to represent a $\frac{1}{2}$ -acre lot and find the solution. (10 lots)
5. Have students write a problem for 4 divided by $\frac{1}{6}$ and then solve, using pattern blocks and graph/grid paper.
6. Have students write a general rule for dividing a whole number by a unit fraction.

Note: If needed, the activity may be stopped here and briefly reviewed the next day before continuing.

7. Have small groups of students use pattern blocks to model the following problem and then represent it on graph paper: “Mark has four packs of paper and wants to repackage them for his Boy Scout project into packs that are each $\frac{2}{3}$ the size of each original pack. How many new packs will he have?”

- a. Students may solve this problem by using four hexagonal blocks to represent the four original packs of paper. They may then use two rhombi to represent $\frac{2}{3}$ of a hexagonal block and find the solution. (six packs)
- b. Students may represent the problem on graph paper as in the following model:



$$4 \div \frac{2}{3} = 6$$

8. Have small groups of students use pattern blocks to model the following problem and then represent it on graph paper: “For a science experiment, a class wants to cut six yards of yarn into $\frac{2}{3}$ -yard pieces. How many pieces will they get?”
9. Have small groups of students write a problem for $8 \div \frac{2}{3}$, use pattern blocks to model it, and then represent it on graph paper to solve.
10. Have students write a general rule for dividing a whole number by $\frac{2}{3}$.
11. Have the students solve the following problems, using the procedures already established:

$$3 \div \frac{3}{4}$$

$$6 \div \frac{3}{4}$$

$$12 \div \frac{3}{4}$$

12. Have students write a general rule for dividing a whole number by $\frac{3}{4}$. Ask: “Why do you have to divide by three?” “Why multiply by four?”
13. Have students develop a rule for dividing any whole number by any fraction that is less than one.

Classroom Assessment

During the activity, observe students as you walk around the room and check for understanding. At the end of the activity, students may respond in their math journals to the following prompt: “Describe a rule for dividing a whole number by a fraction. Describe common circumstances in which people divide by fractions.”

Follow-up/Extensions

Students should be encouraged to find examples of dividing by fractions in the real world. Another representation for pattern blocks is using available software. The following Web sites have “virtual” pattern blocks and activities:

http://www.arcytech.org/java/patterns/patterns_j.shtml

http://www.matti.usu.edu/nlvm/nav/frames_asid_169_g_1_t_2.html

http://step.k12.ca.us/community/fractions_institute/flash/pattern_block.html

Homework

If this activity is used over two days, limit the first night’s homework to problems involving the division of whole number by unit fractions. Answers should have whole number answers. Following completion of the activity, any problems assigned involving division of whole numbers by fractions less than one should result in whole-number answers.

Curriculum Connections

Science: (6.1, 6.2) In designing experiments, a whole may need to be divided into fractional parts.

Social Studies: (USH.3b) As the U.S. continues to expand, areas of land are being developed and may need to be divided into fractional parts.

Egg-Carton Addition

Reporting Categories

Computation and Estimation, Number and Number Sense

Overview

Students will model the addition of fractions, using egg cartons as manipulatives.

Related Standards of Learning 4.2, 4.9, 5.7, 6.6

Approximate Time 25 minutes

Objectives

- Students will review part-whole interpretations of fractions, using egg cartons.
- Students will model real-life situations.
- Students will model the sum of two fractions.

Prerequisite Understandings/Knowledge/Skills

- Understand the part-whole interpretation of fractions
- Understand the concept of addition as an approach to a real-life problem

Materials Needed

- Three egg cartons for each student or pair of students
- Paper and pencils
- Plastic eggs, colored tiles, counters, cubes, or other manipulatives to use as “eggs”
- Multiple copies of the handouts “Egg-Carton Record Sheet” and “Egg-Carton Problem Sheet” for each pair
- Overhead transparency of the “Egg-Carton Record Sheet”
- Overhead markers

Instructional Activity

1. *Initiating Activity:* Model the activity for the class before having them do it on their own. Students should work in pairs or groups to build the first fraction in one carton and the next fraction in the second carton. They then transfer the eggs to a third carton (or to one of the two cartons) in order to interpret the sum.
2. Ask the students to fill $\frac{1}{4}$ of the first carton and $\frac{1}{6}$ of the second carton with “eggs.” Then ask them to transfer all the eggs to another carton and determine the fraction that they have. Five of the 12 cups should be filled, which is $\frac{5}{12}$. Model this for the class, and represent it symbolically as well.

$$\frac{1}{4} + \frac{1}{6} =$$

$$\frac{3}{12} + \frac{2}{12} = \frac{5}{12}$$

3. Ask the students to continue with similar problems, as follows:

a. Find the egg-carton sum of $\frac{3}{4}$ and $\frac{1}{3}$.

b. Find the egg-carton sum of $\frac{2}{3}$ and $\frac{1}{4}$.

c. Find the egg-carton sum of $\frac{1}{2}$ and $\frac{1}{3}$.

4. Ask students to model for the class and record their work on their “Egg-Carton Record Sheets.”
5. Model the problem $\frac{5}{6} + \frac{1}{4}$ for the class. Ask questions. They should realize the sum is greater than 1.
This is a good time to discuss mixed numbers and improper fractions.
6. Give each group a copy of the “Egg-Carton Problem Sheet” and additional copies of the record sheet. Ask them to model each problem situation with their egg cartons and then record their computations on a record sheet. *Note:* The answers to the “Egg-Carton Problem Sheet” are found below.
7. *Closing Activity:* Make transparencies of the “Egg-Carton Record Sheet.” Allow each group to model at least one of its problems for the class.

Follow-up/Extensions

Ask students to make up similar problem situations, solve them, and pose them to other groups.

Answers to “Egg-Carton Problem Sheet”

1. The recipe for pound cake calls for $\frac{2}{3}$ of a dozen eggs. Jane wants to make two pound cakes. Model the eggs needed. ($\frac{4}{3}$ of a dozen or $1\frac{1}{3}$ dozen equal 16 eggs.)
2. It takes $\frac{1}{4}$ of a dozen eggs to make a batch of brownies. Elizabeth has $\frac{5}{6}$ of a dozen eggs left. How many batches can she make? ($\frac{1}{4}$ of a dozen = 3 eggs, and $\frac{5}{6}$ of a dozen = 10 eggs. Therefore, she can make three batches with $\frac{3}{4}$ dozen, or 9 nine eggs, and have one egg left.)
3. Mike bought four dozen eggs at the store. When he returned home he discovered that $\frac{1}{4}$ of the eggs were broken in the first two cartons, $\frac{1}{3}$ of the eggs were broken in the third carton, and $\frac{1}{6}$ of the eggs were broken in the fourth carton. How many dozen eggs was Mike able to save? (The total number broken is $\frac{12}{12}$ or one dozen eggs; thus three of the four dozen remain.)
4. There are two kitchens in the home economics wing of the middle school. The first kitchen has $\frac{2}{3}$ of a case of soft drinks left over (each case holds 24 drinks). The second kitchen has $\frac{3}{4}$ of a case of drinks left. The home economics class has 30 thirsty students. Do you think there are enough drinks? Justify your answer. ($\frac{2}{3}$ of a case equals 16 drinks, and $\frac{3}{4}$ of a case equals 18 drinks. For the total of 34 drinks here, the “whole” is 24, not 12.)
5. Jane bought a dozen eggs at the store for her mother. She scrambled five of them for breakfast, used $\frac{1}{6}$ of a dozen for cookies, and boiled $\frac{1}{3}$ of a dozen to make egg salad sandwiches. Are there any eggs left? If so, how many? (She scrambled five eggs, used two eggs for cookies, boiled four eggs; thus she used a total of 11 eggs, and one egg remains.)

Egg-Carton Record Sheet

Egg-Carton Problem Sheet

For each problem below, make a model using your egg cartons and “eggs.” Then record your solution on the “Egg-Carton Record Sheet.” On a separate sheet of paper, list all the observations you and your partners can make about each situation and the mathematical relationships that you observe. In some cases you may even choose to extend the problem.

1. The recipe for pound cake calls for $\frac{2}{3}$ of a dozen eggs. Jane wants to make two pound cakes. Model the eggs needed.
2. It takes $\frac{1}{4}$ of a dozen eggs to make a batch of brownies. Elizabeth has $\frac{5}{6}$ of a dozen eggs left. How many batches can she make?
3. Mike bought four dozen eggs at the store. When he returned home he discovered that $\frac{1}{4}$ of the eggs were broken in the first two cartons, $\frac{1}{3}$ of the eggs were broken in the third carton, and $\frac{1}{6}$ of the eggs were broken in the fourth carton. How many dozen eggs was Mike able to save?
4. There are two kitchens in the home economics wing of the middle school. The first kitchen has $\frac{2}{3}$ of a case of soft drinks left over (each case holds 24 drinks). The second kitchen has $\frac{3}{4}$ of a case of drinks left. The home economics class has 30 thirsty students. Do you think there are enough drinks? Justify your answer.
5. Jane bought a dozen eggs at the store for her mother. She scrambled five of them for breakfast, used $\frac{1}{6}$ of a dozen for cookies, and boiled $\frac{1}{3}$ of a dozen to make egg salad sandwiches. Are there any eggs left? If so, how many?

Egg-Carton Fractions

Reporting Category

Number and Number Sense

Overview

Students will use an egg carton to model and understand the part-whole concept of fractions.

Related Standards of Learning

4.2, 4.3

Approximate Time

30 minutes

Objectives

- Students will represent fractions by dividing an egg carton (whole) into equal-sized parts (denominator) and filling the specified sections (numerator) with plastic eggs or counters.
- Students will compare egg-carton fractions to determine equivalence.

Prerequisite Understandings/Knowledge/Skills

- Understand that a fraction is a way of representing part of a unit whole
- Understand that the denominator tells the number of equal-sized parts in a whole
- Understand that the numerator tells the number of equal-sized parts being considered

Materials Needed

- An egg carton for each student or pair of students
- Yarn or string cut in lengths of 10-to-12 inches
- Plastic eggs, colored tiles, counters, cubes, or other manipulatives to use as “eggs”
- Multiple copies of the handout “Egg-Carton Record Sheet” for each pair of students

Instructional Activity

1. *Initiating Activity:* Distribute egg cartons, “eggs,” and six lengths of yarn to each student or pair of students. Ask them to model the fraction $\frac{1}{2}$ with their materials. The models will reveal ways in which the students perceive the meaning of fractions. Ask them to show their models and explain their reasoning. Some of the responses may include the following:
 - They may divide the egg carton into two equal-sized parts and fill one of the parts with eggs. This is the “part-whole” concept of fractions, which this activity reinforces.
 - They may place six eggs in the carton, but explain it as putting in “one egg for every two spaces.” This is a “ratio” concept of fractions.
 - They may select a whole that is smaller than 12 spaces and fill one-half of this different whole — e.g., select four spaces as the whole and fill two of them. This is not invalid as long as they clearly understand the whole and part.
 - They may concentrate on finding one-half of the total number of eggs and may fill *any* six of the 12 spaces in the carton.
 - They may demonstrate their misconceptions with incorrect models — e.g., they may divide the carton into two parts that are *not* equal and fill one of the parts.
2. Clarify student perceptions by modeling a fraction and then having the students model other fractions. Ask each pair to model $\frac{3}{4}$ by dividing the carton into four equal-sized parts with yarn, and filling three of the parts with eggs or cubes.
3. Use the part-whole concept to complete the activity. In an egg-carton fraction, the denominator represents the number of equal-sized parts or divisions of the egg carton (marked with yarn), and the numerator represents the number of these parts that should be filled with eggs.

4. Ask each pair of students to model $\frac{2}{3}$ and $\frac{4}{6}$ in their cartons. Using the models, ask the following questions about both fractions:
 - a. “How many eggs represent the ‘whole’ carton?” (12)
 - b. “When you modeled $\frac{2}{3}$ of the carton, how many eggs did you use?” (8)
 - c. “When you modeled $\frac{4}{6}$ of the carton, how many eggs did you use?” (8)
 - d. “How do the fractions $\frac{2}{3}$ and $\frac{4}{6}$ compare?” (They are equivalent.)

Depending on the prior experience of your class, call for responses, or lead the students to understand that *as long as the entire carton represents the whole*, fractions that have the same number of egg cups filled will be equivalent.

5. Distribute copies of the “Egg-Carton Record Sheet,” and ask students to model and record (draw) all of the possible fractions that can be made from a dozen eggs. Each fraction should be listed in all of its equivalent forms. The fractions that you might expect students to list are shown in the chart below.
6. *Closing Activity:* Call for answers, and allow students to ask questions. Model any fraction that creates discussion. Clear up misconceptions if they exist.

Classroom Assessment

Examine the students’ record sheets either as the students are drawing them or at the conclusion of the activity. Use them to pinpoint and address individual weaknesses.

Follow-up/Extensions

The egg cartons can be used to solve real-life problems and to add or subtract with fractions.

Number of Eggs Filled	Equivalent Fractions Formed					
0	$\frac{0}{1}$	$\frac{0}{2}$	$\frac{0}{3}$	$\frac{0}{4}$	$\frac{0}{6}$	$\frac{0}{12}$
1	$\frac{1}{12}$					
2	$\frac{1}{6}$			$\frac{2}{12}$		
3	$\frac{1}{4}$			$\frac{3}{12}$		
4	$\frac{1}{3}$		$\frac{2}{6}$		$\frac{4}{12}$	
5	$\frac{5}{12}$					
6	$\frac{1}{2}$	$\frac{2}{4}$		$\frac{3}{6}$		$\frac{6}{12}$
7	$\frac{7}{12}$					
8	$\frac{2}{3}$		$\frac{4}{6}$		$\frac{8}{12}$	
9	$\frac{3}{4}$			$\frac{9}{12}$		
10	$\frac{5}{6}$			$\frac{10}{12}$		
11	$\frac{11}{12}$					
12	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{6}{6}$	$\frac{12}{12}$

Egg-Carton Record Sheet

Four in a Row

Reporting Category
Overview

Computation and Estimation
Students will add and subtract fractions with fraction strips in a game format.

Related Standards of Learning 4.9, 5.7, 6.6

Approximate Time 20 minutes

Objective

Students will add and subtract common fractions, using fraction strips.

Prerequisite Understandings/Knowledge/Skills

- Understand how to model fractions with fraction strips
- Understand how to add and subtract fractions with fraction strips

Materials Needed

- Fractions strips (made in the “Fraction Strips” activity)
- Copy of the handouts “Fraction Chart” and “Four-in-a-Row Game Board” for each group
- Beans of two colors or other board markers

Instructional Activity

1. *Initiating Activity:* Model the game for the class.
2. Let each pair of students decide who goes first. Player One chooses two fractions on the “Fraction Chart” that can be added or subtracted to get one of the answers on the “Four-in-a-Row Game Board.” Player One must first demonstrate the problem with the fraction strips (or another method), after which he/she may cover the answer with a bean or marker.
3. Each fraction may be used only once. Put a bean on each fraction as it is used.
4. Player Two now takes a turn. Play continues until someone covers four fractions in a row — horizontally, vertically, or diagonally.

Fraction Chart

$\frac{7}{8}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{3}$
$\frac{2}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{2}{4}$
$\frac{1}{8}$	$\frac{5}{8}$	$\frac{3}{6}$	$\frac{6}{8}$
$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{8}$	$\frac{1}{5}$
$\frac{9}{10}$	$\frac{7}{12}$	$\frac{3}{10}$	$\frac{5}{12}$

Four-in-a-Row Game Board

$\frac{1}{4}$	$\frac{3}{8}$	$\frac{2}{3}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{1}{12}$	$\frac{7}{8}$	$\frac{1}{8}$
$\frac{5}{8}$	$\frac{1}{12}$	$\frac{11}{8}$	$\frac{3}{2}$
$\frac{13}{10}$	$\frac{9}{8}$	$\frac{5}{4}$	0

Fraction Strips

Reporting Category

Number and Number Sense

Overview

Students will fold and label equal length strips of paper to make individual sets of fraction manipulatives. While the primary grade level for this activity is fourth, it can be enriched and extended for other grade levels.

Related Standards of Learning 4.2, 4.3, 4.9, 6.4

Approximate Time 45 minutes

Objectives

- Students will create their own set of fraction strips by cutting strips into specific parts:

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{12}$$

- Students will visually compare fractions, using fraction strips.

Notes: Omit $\frac{1}{6}$ and $\frac{1}{12}$ if this activity is used in third grade. The fraction $\frac{1}{10}$ will be addressed in lessons on decimals.

Prerequisite Understandings/Knowledge/Skills

- Understand that a fraction is a way of representing part of a unit whole
- Understand that the denominator tells the number of equal-sized parts in a whole
- Understand that the numerator tells the number of equal-sized parts being considered

Note: If students do not have the prerequisite understandings, knowledge, and/or skills, refer back to previous grade-level Standards of Learning and lessons for ideas to reteach. The language of numerator and denominator is encouraged in fourth, fifth, and sixth grades.

Materials Needed

- Seven equal-sized strips of different-colored construction paper for each student
- Scissors
- Pencils or markers
- Sticky notes
- Chart paper
- A letter-size, three-hole-punched envelope for each student to hold strips for future projects

Note: For easier management, put the set of seven paper strips, a pair of scissors, and a marker in a plastic storage bag for each student.

Instructional Activity

1. *Initiating Activity:* Brainstorm with the students what ideas they have about fractions — what they remember from previous grades or from experiences outside of school. Have students record their ideas on sticky notes and place them on a sheet of chart paper. (Students may put their initials on the back of their sticky notes, and these may be used later for individual assessment of prior knowledge.) Accept all responses. At the end of the activity, students may examine each idea and verify whether they are correct or incorrect.
2. Distribute seven strips of colored construction paper, a pair of scissors, and a marker to each student. Explain that they will be creating their own set of fraction strips with the materials. As you work through the steps below, model each step with your own strips.

3. Ask students to label one strip “1”.
4. Have the students fold a second strip in half. Talk about the two equal-sized parts, demonstrating by folding that the two parts are congruent. Have students label each part “ $\frac{1}{2}$ ” and then cut on the fold. Ask the students how many of the $\frac{1}{2}$ strips it will take to cover the 1 strip. Have them place the two $\frac{1}{2}$ strips on the 1 strip to prove their answer.
5. Model the same procedures for fourths, choosing a different-colored strip. At each step, discuss how many of the $\frac{1}{4}$ strips it takes to cover the 1, and how many of the $\frac{1}{4}$ it takes to cover the $\frac{1}{2}$. After modeling each step for the students, have them repeat the step.
6. Model the same procedures for eighths, choosing a different-colored strip. At each step, talk about how many of the $\frac{1}{8}$ strips it takes to cover the 1, how many of the $\frac{1}{8}$ it takes to cover the $\frac{1}{4}$, and how many of the $\frac{1}{8}$ it takes to cover the $\frac{1}{2}$. After modeling each step for the students, have them repeat the step.
7. Model the same procedures for thirds, choosing a different-colored strip. This step takes a little more practice to fold the strip into three equal-sized parts, or students may use a ruler to mark the strip into thirds. At each step, talk about how many of the $\frac{1}{3}$ strips it takes to cover the 1, or whole. Ask how many of the $\frac{1}{3}$ strips it will take to cover the $\frac{1}{2}$. Students will discover that thirds will not cover the half exactly. After modeling each step for the students, have them repeat the step.
8. At fourth grade and above, model the same procedures for sixths, choosing a different-colored strip. Model the procedures used to make thirds, and then fold the strip in half and cut on the folds to get six equal-sized parts. Label each part “ $\frac{1}{6}$.” Ask the students how many $\frac{1}{6}$ strips it will take to cover the 1, the half, and the fourth. After modeling each step for the students, have them repeat the step. Students will discover that sixths will not cover the fourth exactly.
9. At this point, you may want the students to explore twelfths and share with their partners how they would create the $\frac{1}{12}$ strips.
10. Have the students write their name on all their fraction strips, place them in their three-hole-punched envelope, and store in their binders.

Note: This activity should help the students visualize fractions and begin to conceptualize the relationships between the common fractions. The process of folding and cutting, however, is not an exact one, and the handmade strips may lead to incorrect comparisons.

11. *Closing Activity*: Refer back to the sticky notes from the beginning of the activity. Help students reflect on the concepts that were demonstrated in the activity. Ask students to verbalize the concepts learned in this activity, including pictorial representations, if applicable. The concepts should include:
- A fraction is a way of representing part of a unit whole.
 - A fraction is used to name a part of one thing.
 - Wholes can be broken into equal-sized parts, and the parts can be reassembled into wholes.
 - Equal-sized parts have special names: *halves*, *thirds*, *fourths* or *quarters*, *sixths*, *eighths*, *twelfths*.
- Organize the sticky notes on a wall chart by having the students indicate in which of three columns (Correct, Incorrect, Need More Information) the ideas should be placed.

Classroom Assessment

During the activity, observe students as you walk around the room and check for understanding. At the end of the activity, students may respond to the following prompts in their math journals:

- “What did you notice that was the same when you created the $\frac{1}{2}$ and the $\frac{1}{8}$ strips?”
- “What did you notice that was different when you created the $\frac{1}{2}$ and the $\frac{1}{8}$ strips?”

To see if students understand the concept that fractions divide areas into equal-sized parts, ask, “If you fold your strips to create $\frac{1}{5}$, how do you know they are fifths?” (There should be 5 equal-sized parts.)

Follow-up/Extensions

These individual strips can be used in numerous activities for fractions. Suggested activities include comparing fractions, finding equivalent fractions, and adding and subtracting fractions. See additional activities in this document.

Homework

Share the strips with your family, and explain the math lessons that you learned today.

Curriculum Connections

Science: “Customary Rulers,” “Magnified Inch” (*Family Math*, p. 86)

Social Studies: Money, map reading

Literature: Pallotta, Jerry. *The Hershey’s Milk Chocolate Fraction Book*, Scholastic, 1999.

Fraction-Strip Addition

Reporting Category

Computation and Estimation

Overview

Students will work together to add two fractions, using their sets of fraction strips (or fraction tiles).

Related Standards of Learning 4.2, 4.9, 5.7, 6.6

Approximate Time

30 minutes

Objective

Students will add two fractions, using their sets of fraction strips, and interpret the answers in equivalent forms.

Prerequisite Understandings/Knowledge/Skills

- Understand that a fraction is a way of representing part of a unit whole
- Understand how to model a fraction with a manipulative

Materials Needed

- Individual sets of fraction strips (made in the “Fraction Strips” activity or made from the fraction strip sheet templates on the following pages)
- Copies of the handout “Fraction Sum Sheet”

Note: Copy each template of fraction strip sheets onto a different color of cardstock or paper. If this activity is used in grade four, be sure to precede the activity with the “Fraction Strips” activity. Students who experience difficulty with the unlike denominators may need additional examples (steps 2c and 2d).

Instructional Activity

1. *Initiating Activity:* Each student should have a complete set of fraction strip sheets, one each for the whole (1 unit), halves, thirds, fourths, sixths, eighths, tenths, and twelfths. These can be overlapped to work problems and will probably be easier to use in this format. If a student does not understand the overlapping process, the strip sheets may be cut apart.
2. Use the “Fraction Sum Sheet” to model several problems for the class, for example:
 - a. Say, “Add the fractions $\frac{1}{2}$ and $\frac{1}{4}$ on your unit strip. What is the sum?” ($\frac{3}{4}$)
 - b. Ask, “How many $\frac{1}{8}$ pieces does it take to cover your answer?” (6) Ask, “What is another way to express $\frac{3}{4}$ as a fraction?” ($\frac{6}{8}$)
 - c. Say, “Add $\frac{3}{4}$ and $\frac{3}{8}$. What do you think is the sum?” Students may need help at this point, seeing that the answer is more than one. Encourage them to put two whole cards together and continue the same process. They should be able to tell you that the answer is equivalent to 9 eighths, or to one whole and one eighth.
 - d. Say, “Add $\frac{2}{3}$ and $\frac{3}{4}$. What is the sum?” Students may not be able to give this answer as neither thirds nor fourths will fit exactly. Encourage them to experiment with their other fraction strips until someone discovers that twelfths will work. Make sure that all groups understand that 17 twelfths, or one whole and 5 twelfths, will fit this sum exactly. On a transparency, model $\frac{2}{3}$ as $\frac{8}{12}$

and $\frac{3}{4}$ as $\frac{9}{12}$. Transparent copies of the fraction strips might help to do this. Do not mention rules for addition unless students bring them up. The emphasis here should be on the physical visualization of the fraction sum and the visualization of an equivalent sum.

3. Give each pair of students a copy of the handout “Fraction Sum Sheet” and several of the following fraction exercises. Alternatively, you might give them a handout of all eight problems or write the problems on the board one at a time. Ask them to work with their fraction strips to find the sums and to record the equivalent fractions they used in the process. (See answers below.)

a. $\frac{5}{12} + \frac{1}{3} = \underline{\hspace{2cm}}$ b. $\frac{2}{3} + \frac{1}{4} = \underline{\hspace{2cm}}$ c. $\frac{5}{8} + \frac{7}{8} = \underline{\hspace{2cm}}$
d. $\frac{3}{4} + \frac{5}{6} = \underline{\hspace{2cm}}$ e. $\frac{7}{10} + \frac{1}{2} = \underline{\hspace{2cm}}$ f. $\frac{5}{6} + \frac{2}{3} = \underline{\hspace{2cm}}$

Reserve these problems for grades 5 through 8:

g. $1\frac{1}{4} + 2\frac{2}{3} = \underline{\hspace{2cm}}$ h. $3\frac{1}{3} + 1\frac{5}{6} = \underline{\hspace{2cm}}$

4. *Closing Activity:* Ask each group to discuss the strategies they used, and look for patterns. Write the problem $\frac{3}{4} + \frac{7}{8}$ on the board or overhead. Ask each student to find the answer and write a short paragraph justifying the answer. Encourage drawings or diagrams. Call for responses from students, or allow each group time to discuss their answers.

Answers to the Fraction Exercises

a. $\frac{9}{12}$ or $\frac{3}{4}$ b. $\frac{11}{12}$ c. $\frac{12}{8}$ or $\frac{3}{2}$ or $1\frac{1}{2}$
d. $\frac{19}{12}$ or $1\frac{7}{12}$ e. $\frac{12}{10}$ or $\frac{6}{5}$ or $1\frac{1}{5}$ f. $\frac{9}{6}$ or $\frac{3}{2}$ or $1\frac{1}{2}$
g. $3\frac{11}{12}$ h. $5\frac{1}{6}$

Fraction Sum Sheet

Fraction **Denominators**

	=	

+

	=	



Sum



Fraction Strips — One Whole

1
1
1
1
1

Fraction Strips — Halves

$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$

Fraction Strips — Fourths

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Fraction Strips — Eighths

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Fraction Strips — Tenths

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Fraction Strips — Thirds

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Fraction Strips — Sixths

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Fraction Strips — Twelfths

$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

Fraction-Strip Subtraction

Reporting Category

Computation and Estimation

Overview

Students will subtract fractions from a whole unit, using their sets of fraction strips.

Related Standards of Learning 4.3, 4.9, 5.7, 6.6

Approximate Time

30 minutes

Objectives

- Students will subtract fractions, using fraction strips.
- Students will recognize equivalent forms of fractions, using fraction strips.

Prerequisite Understandings/Knowledge/Skills

- Understand how to model a fraction with fraction strips
- Understand how to model equivalent fractions with fraction strips

Materials Needed

- Individual sets of fraction strips
- Fraction spinner or random fraction cards from the activity “Which Is Closer?”

Instructional Activity

1. *Initiating Activity:* Give each student a complete set of fraction strips. Hold up the unit (1) strip and ask, “What does this represent? (one unit or whole) Ask, “How many fourths make up this unit? (4) Say, “Place the unit and the four fourths from your set of strips in front of you on the desk. If this whole unit represents a candy bar, and I give $\frac{1}{4}$ of it to Sue, how much of the candy bar remains?” ($\frac{3}{4}$) Then ask, “If I give an additional $\frac{3}{8}$ of the candy bar to Joe, how much of the original candy bar will be left for me?” Encourage students to explore with the one-eighth strips to discover this answer. ($\frac{3}{8}$)
2. Once the students understand the process of using fraction strips to subtract from a whole unit, group the students into pairs or small groups to play the game “Take One.” Give each group a set of fraction cards from the “Which Is Closer?” activity or other source. Model the activity for the class, using overhead fraction strips. Start with the unit piece. Subtract $\frac{1}{3}$. Ask for the result. Subtract $\frac{1}{4}$ from that answer, using twelfths pieces. Interpret the answer. Answer any questions that arise.

DIRECTIONS FOR THE GAME “TAKE ONE”

- a. Begin with the whole unit or two halves. Draw a fraction card from the center of table and remove the amount shown from the whole or two halves.
- b. Keep track of what is left with the fraction strips.
- c. Substitute equivalent fractions as needed.
- d. Alternate turns, with each student drawing a card and subtracting.
- e. The first player with a blank board wins.

3. Ask the pairs of students to place in front of them on the desk fraction strips to represent $\frac{7}{8}$ and $\frac{1}{2}$. Ask, “Which is bigger?” ($\frac{7}{8}$) Ask them, “How much bigger?” Following their previous work with addition of

strips, pairs may be able to line up the two fractions one under the other and fit fraction strips to represent the difference. Call on volunteers to model their work. If necessary, model the problem yourself on the overhead with transparent fraction strips to show that the difference is $\frac{3}{8}$.

4. Ask the groups to model the following problems one at a time and record their models by drawing on paper. Have them demonstrate and explain their correct solutions on the overhead projector.

a. $\frac{2}{3} - \frac{1}{4} = \underline{\hspace{2cm}}$ b. $\frac{3}{4} - \frac{5}{8} = \underline{\hspace{2cm}}$

Reserve these problems for grades 5 through 8:

c. $1\frac{2}{3} - \frac{1}{6} = \underline{\hspace{2cm}}$ d. $1\frac{1}{2} - \frac{3}{4} = \underline{\hspace{2cm}}$

e. $1\frac{5}{8} - \frac{3}{4} = \underline{\hspace{2cm}}$

Students may experience difficulty with problems such as d. and e. in which it is necessary to regroup, breaking the unit (1) into smaller parts. Model this process for them until they understand this concept.

5. *Closing Activity:* Ask each student to model a solution to the following problem, record a diagram or picture of their model, and write an explanation of their solution: “Brad has $\frac{3}{4}$ of a pound of fudge, and Julie has $\frac{7}{8}$ of a pound. Together do they have enough fudge to serve 12 people $\frac{1}{8}$ of a pound of fudge each?” Solutions will vary in appearance, but all students should come to an understanding that Brad and Julie have a total of $\frac{13}{8}$ pounds of fudge — enough to serve 13 people $\frac{1}{8}$ of a pound each.

Follow-up/Extensions

The activities and problems in this activity can be done quite easily with fraction squares or circles, with egg cartons, or with other fraction manipulatives. The use of a ruler is important in science for making measurements. This activity could be tied to subtracting lengths — given in halves, fourths, and eighths of an inch or tenths of a centimeter — from a starting length. For example, groups might be given a yard or meter of adding machine tape and asked to cut as many $6\frac{3}{4}$ -in. or $5\frac{7}{10}$ -cm lengths as possible from it, recording the result at each step. This would reinforce subtraction as well as set the stage for division. Each group could cut lengths of a different size and make comparisons.

Curriculum Connections

Science: Measurement

Fraction Riddles

Reporting Categories

Number and Number Sense, Computation and Estimation

Overview

This activity can be used *after* adding and subtracting fractions, using fraction strips, as reinforcement for both processes. This is *not* intended to be a complete lesson.

Related Standards of Learning 4.2, 4.9, 5.7, 6.6

Approximate Time 15 minutes

Objectives

- Students will estimate and compare fraction values in order to make wise choices of numbers used in solving riddles.
- Students will add or subtract two fractions, using their sets of fraction strips.

Prerequisite Understandings/Knowledge/Skills

- Understand how to add and subtract fractions, using fraction strips or other manipulatives

Materials Needed

- Individual sets of fraction strips
- A copy of the handout “Fraction Chart” for each pair of students

Instructional Activity

1. *Initiating Activity:* Give each pair of students fraction strips and a copy of the handout “Fraction Chart.” Pose the following riddles to the class one at a time. Ask the pairs of students to find as many *different* solutions to the riddles as they can, using only fractions from the “Fraction Chart.” Display responses on the overhead or chart paper.

RIDDLES

Riddle 1: I have two fractions whose sum is more than $\frac{1}{2}$ but less than 1. What two fractions from the chart might I have?

Riddle 2: I have two fractions whose difference is less than $\frac{1}{2}$. What two fractions from the chart might I have?

2. Have each student choose two fractions from the chart and either add or subtract them. Then have each student tell his/her partner the *result* of the addition or subtraction, and have the partner guess the original fractions.
3. Have each student make up a riddle using fractions from the chart and then challenge his/her partner to solve it.

Follow-up/Extensions

To make this more challenging, students could take turns answering each riddle and cross out the fractions as they use them.

Fraction Chart

$\frac{7}{8}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{3}$
$\frac{2}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{2}{4}$
$\frac{1}{8}$	$\frac{5}{8}$	$\frac{3}{6}$	$\frac{6}{8}$
$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{8}$	$\frac{1}{5}$
$\frac{9}{10}$	$\frac{7}{12}$	$\frac{3}{10}$	$\frac{5}{12}$

Gridlock

Reporting Categories

Number and Number Sense; Computation and Estimation; Patterns, Functions, and Algebra

Overview

Students will use proportional reasoning to apply their experience with unit ratio problems about population density to a new situation. They will do this in order to create another example of unit ratio and use that ratio to create a proportional reasoning situation.

Related Standards of Learning

6.2, 7.6, 8.17

Approximate Time

15 minutes

Objective

Students will work with unit ratios and proportional reasoning to solve a problem.

Prerequisite Understandings/Knowledge/Skills

- Understand how to construct a unit ratio
- Understand how to solve proportions
- Understand how to use a calculator

Materials Needed

- A copy of the handout “Gridlock Recording Sheet” for each student
- Calculators

Note: This activity is designed to follow the unit ratio problems about population density found in the “Population Density” and “Virginia Population Density” activities. A brief review/discussion of unit ratios may be helpful before beginning the investigation.

Instructional Activity

1. *Initiating Activity:* Pass out the “Gridlock Recording Sheet,” and pose the following problem: “According to the *Guinness Book of Records*, Hong Kong is reported to have the highest traffic density in the world. In 1992, there were 418 registered cars and trucks per mile of road. Another way to represent this ratio would be to say there are about 12.63 feet of road per registered vehicle in Hong Kong.”
2. Ask how the *Guinness Book of Records* determined these two different unit ratios.
3. Ask what the units are in each statement.
4. *Closing Activity:* Present the students with the following situation: “The city of Beetleville has 450,237 registered vehicles per 3,000 miles of road. What is the traffic density of Beetleville? Be prepared to explain your solution.”
5. Ask the students to calculate both the number of vehicles per mile of road and the number of feet of road per vehicle.

Classroom Assessment

Be certain that the students understand the solutions to the problems as follows:

Hong Kong problem: If there are 418 vehicles per mile of road, then the ratio is $\frac{418 \text{ cars}}{1 \text{ road mile}}$. If I want to convert to feet, I know that 1 mile = 5,280 ft. Therefore, $\frac{418 \text{ cars}}{5,280 \text{ road ft.}} = \frac{1 \text{ car}}{x \text{ road ft.}}$. So $x = \frac{5,280}{418} = 12.63$ road feet per vehicle.

Beetleville problem: The city of Beetleville has $\frac{450,237 \text{ vehicles}}{3,000 \text{ road miles}} = 150.1$ vehicles per road mile. Therefore, $\frac{150.1 \text{ vehicles}}{5,280 \text{ road ft.}} = \frac{1 \text{ vehicle}}{x \text{ road ft.}}$. So, $x = \frac{5,280}{150.1} = 35.18$ road feet per vehicle.

Gridlock Recording Sheet



1. According to the *Guinness Book of Records*, Hong Kong is reported to have the highest traffic density in the world. In 1992, there were 418 registered cars and trucks per mile of road. Another way to represent this ratio would be to say there are about 12.63 feet of road per registered vehicle in Hong Kong. How did the *Guinness Book of Records* determine these two different unit ratios? What are the units in each statement? (For this problem, remember that 1 mile = 5,280 ft.)

2. The city of Beetleville has 450,237 registered vehicles for 3,000 miles of road. What is the traffic density of Beetleville? Calculate both the number of vehicles per mile of road and the number of feet of road per vehicle. Be prepared to explain your method of calculation.

Measuring Cups

Reporting Categories

Number and Number Sense, Computation and Estimation

Overview

Students will determine how many fractional parts of a cup are contained in a specified cup quantity.

Related Standards of Learning 6.6a, 6.7, 7.4, 8.3

Approximate Time

30 minutes

Objectives

- Students will identify the whole and the unit in a given division problem.
- Students will solve expressions involving division of fractions.
- Students will discuss patterns that develop when dividing fractions.
- Students will discuss why the procedure of “invert and multiply” is used.

Prerequisite Understandings/Knowledge/Skills

- Understand that fraction computation uses the same ideas as whole-number computation, and how to apply those concepts to fractional parts
- Understand that an estimated answer helps validate the reasonableness of a computed answer

Materials Needed

- A full set of measuring cups for each pair or small group of students
- Food coloring (if using clear measuring cups)

Instructional Activity

1. *Initiating Activity:* Ask students to develop a word problem for the expression $10 \div 5$. It should take about a minute for them to do this task.
2. Ask a few students to share their word problem with the whole class. Hold a short discussion about the language used in these word problems. Common phrases that may arise are: “Ten items are shared fairly with 5 people.” or “How many 5s are in 10?”
3. Give the students the expression $10 \div 5$. Ask, “What does 10 refer to?” “What does 5 refer to?” When dividing, we always refer to the whole. Thus, in this expression, 10 refers to the whole, and 5 refers to the size of the measuring unit or set.
4. Continue this discussion by giving the students the expression $\frac{3}{4} \div \frac{1}{2}$. Ask, “What does $\frac{3}{4}$ refer to?” “What does $\frac{1}{2}$ refer to?” In this expression, $\frac{3}{4}$ refers to the whole, and $\frac{1}{2}$ refers to the size of the measuring unit or set.
5. Ask students to use the previous example of $10 \div 5$ to determine how they would solve $\frac{3}{4} \div \frac{1}{2}$. It may be necessary to help students make the connection and to tell them that they need to determine how many $\frac{1}{2}$ s are in $\frac{3}{4}$.

6. *Demonstration:* Fill a $\frac{3}{4}$ measuring cup full of water. (Use food coloring in the water if clear measuring cups are available.) Ask students: “How many $\frac{1}{2}$ cups are in this $\frac{3}{4}$ cup of water?” Another way of asking this is: “How many times can you pour the $\frac{3}{4}$ cup of water into the $\frac{1}{2}$ cup?”
7. After several students have given and defended their answer, physically pour water from the $\frac{3}{4}$ cup into the $\frac{1}{2}$ cup. Once the $\frac{1}{2}$ cup is full, ask students: “How many $\frac{1}{2}$ cups are in the $\frac{3}{4}$ cup?” They should recognize that there is *more than* one $\frac{1}{2}$ cup in a $\frac{3}{4}$ cup because there is water remaining in the $\frac{3}{4}$ cup.
8. Pose the question: “What should we do with the remaining water?” At this point, someone may make the common mistake of saying that because $\frac{1}{4}$ of a cup of water remains, there are one-and-one-fourth $\frac{1}{2}$ s in $\frac{3}{4}$. Students should physically pour the remaining water into the $\frac{1}{2}$ cup. They should then recognize that the remaining water ($\frac{1}{4}$ of a cup) fills *half* of the $\frac{1}{2}$ cup.
9. Repeat the beginning question: How many $\frac{1}{2}$ cups are in $\frac{3}{4}$ of a cup? (Solution: There are one and one-half $\frac{1}{2}$ cups in $\frac{3}{4}$ of a cup.) Then, repeat the original question: How many $\frac{1}{2}$ s are in $\frac{3}{4}$? (Solution: There are one and one-half $\frac{1}{2}$ s in $\frac{3}{4}$.)
10. Have students select a partner and solve the following expressions. Allow them to use measuring cups or fraction bars, if available. Partners should discuss the method used to solve each expression.
- $\frac{1}{2} \div \frac{1}{3}$ (How many $\frac{1}{3}$ cups are in $\frac{1}{2}$ of a cup?)
 - $\frac{1}{3} \div \frac{1}{4}$ (How many $\frac{1}{4}$ cups are in $\frac{1}{3}$ of a cup?)
 - $1 \div \frac{2}{3}$ (How many $\frac{2}{3}$ cups are in 1 cup?)
 - $2 \div \frac{3}{4}$ (How many $\frac{3}{4}$ cups are in 2 cups?)
 - $\frac{3}{4} \div \frac{1}{3}$ (How many $\frac{1}{3}$ cups are in $\frac{3}{4}$ of a cup?)
 - $1\frac{1}{3} \div \frac{1}{2}$ (How many $\frac{1}{2}$ cups are in $1\frac{1}{3}$ cups?)
 - $2\frac{1}{4} \div \frac{1}{3}$ (How many $\frac{1}{3}$ cups are in $2\frac{1}{4}$ cups?)
 - $1\frac{1}{2} \div \frac{1}{3}$ (How many $\frac{1}{3}$ cups are in $1\frac{1}{2}$ cups?)
 - $1\frac{3}{4} \div \frac{3}{4}$ (How many $\frac{3}{4}$ cups are in $1\frac{3}{4}$ cups?)
 - $1\frac{2}{3} \div \frac{1}{4}$ (How many $\frac{1}{4}$ cups are in $1\frac{2}{3}$ cups?)

11. As a class, verify the results of each expression.
12. Have the students continue to work with their partners to solve the following expressions. Discuss the patterns that develop. As a class, verify the results of each expression.

a. $1 \div \frac{2}{5}$

b. $2 \div \frac{2}{5}$

c. $3 \div \frac{2}{5}$

d. $4 \div \frac{2}{5}$

Note: Proportional reasoning indicates that there will be twice as many two-fifths in the second expression than in the first. Similarly, there will be three times as many two-fifths in the third expression than in the first. Also, students may notice that you can multiply the dividend by the denominator of the divisor and then divide this solution by the numerator of the divisor to arrive at the correct solution. Be certain to explain that the reason this method works is because division is the reciprocal of multiplication. Therefore, when we divide, it is the same as multiplying by the reciprocal. This notion helps to explain why we “invert and multiply.”

Classroom Assessment

While the pairs are working, circulate and listen to the discussions taking place. Watch how students are selecting a strategy or approaching the problem. If anyone is simply falling back on the “invert and multiply” algorithm, insist that they show you they understand the problem and its solution with concrete materials or in narrative form.

Follow-up/Extensions

Write about how you would solve the following problem: A recipe calls for $1\frac{3}{4}$ cups of butter. Each stick of butter is $\frac{1}{2}$ of a cup. How many sticks of butter will be needed for this recipe?

Museum Walk

Reporting Category

Number and Number Sense

Overview

Students will discuss common uses of fractions, decimals, and percent and their meanings. Using a museum-walk model, groups will rotate around the room and share findings with one another.

Related Standard of Learning

6.1, 7.1

Approximate Time

30 minutes

Objectives

- Students will discuss and interpret common uses and meanings of fractions, decimals, and percent.
- Students will develop an understanding that the same quantity can be expressed as a fraction, decimal, or percent.

Materials Needed

- Chart paper
- Colored markers of 3 to 6 different colors.

Instructional Activity

1. *Initiating Activity:* Make three charts, headed as follows: “Real-Life Uses of Fractions,” “Real-Life Uses of Decimals,” and “Real-Life Uses of Percent.” Model the use of the charts by listing some examples of ways we use these kinds of numbers. Divide the students into three groups, and assign one group to each chart. If dividing the class into three groups would make the groups too large, then make two charts of each type and have six groups, two groups per each type of chart.
2. Give each group a different colored marker. Have the groups rotate to the three different charts to record their uses.
3. Give the students approximately five minutes at each station. Continue until the groups are back to their starting places.
4. After walking and recording, ask students to compare the lists. Ask questions such as: “In what ways are fractions, decimals, and percents alike?” “In what ways are they different?” Compare answers and lists.
5. *Closing Activity:* Write the fraction “one-half” on the board in the three notations: $\frac{1}{2}$, 0.5, 50%. Ask:
 - a. What do you know about these three numbers?
 - b. How are they the same?
 - c. How are they different?
 - d. Where do you see these numbers on our charts?

Classroom Assessment

Listen to student discussions about the use of fractions, decimals, and percents. In certain cases, one representation may be more commonly used than others. For example, a hitter can have a .265 batting average, but we do not say he has a “265 thousandth” average. In some situations, one form may not make sense to us. For example, we say “ $\frac{1}{2}$ inch” but not “50% of an inch.” Sale prices may be 25% or $\frac{1}{4}$ off, but not 0.25 off. A jogger runs 3 and $\frac{1}{2}$ miles, but not 3 and 50% miles. The language of their answers and comparisons of different uses is an important discussion to assess understanding of the representations of rational numbers.

Follow-up/Extensions

Students may make a collage of fractions, decimals, and percents that they find in print media. The “Real-Life Uses” may also be extended following future activities.

Paper-Folding Multiplication

Reporting Categories	Number and Number Sense, Computation and Estimation
Overview	Students will participate in paper-folding activities that will serve as models for conceptualizing multiplication of fractions.
Related Standards of Learning	4.2, 4.3, 6.4
Approximate Time	45 minutes

Objectives

- Students will use paper folding to visualize the meaning of multiplication of fractions.
- Students will compare fractions.

Prerequisite Understandings/Knowledge/Skills

- Understand and use the terms *fraction*, *whole*, *numerator*, and *denominator*
- Understand that the denominator tells the number of equal-size parts in a whole
- Understand that the numerator tells the number of equal-sized parts being considered

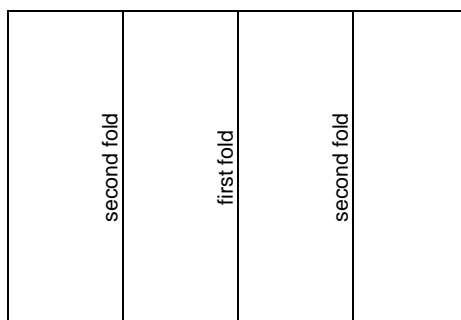
Materials Needed

- Plain copy paper
- Blank transparency sheets
- Transparency of “Paper-Folding Problems”
- Overhead and regular markers
- Crayons/colored pencils
- One large, sectioned candy bar for demonstration

Instructional Activity

1. *Initiating Activity*: It is important to script this activity carefully and thoroughly for the students. The value of this activity lies in clearly understanding the relationship between the paper folds and the operation of multiplication. Each step of the activity should be fully discussed. Lead the students in the following dialogue and steps: “Today we are using an area model for fractions. We will think of the area of a standard sheet of paper as one unit of area. Now fold your sheet of paper by pressing the shorter edges together. This is called a ‘hamburger fold.’ What is the area of each part?” Encourage students to respond that the area of each part is $\frac{1}{2}$ of the unit. Say, “As we can all see by our folds, we have the result of taking $\frac{1}{2}$ of 1 unit, so each part is now $\frac{1}{2}$. Color *one* of your $\frac{1}{2}$ units.”
2. Then say, “Fold the paper again by pressing the shorter edges together — another hamburger fold. What do you have now?” Call for responses, and discuss them fully. Make sure that the class understands that you have taken $\frac{1}{2}$ of ($\frac{1}{2}$ of 1), or $\frac{1}{2}$ of $\frac{1}{2}$. Ask the students to unfold the paper, examine it, and tell you what part of the *original* paper is represented by $\frac{1}{2}$ of the *colored region*. ($\frac{1}{4}$) Therefore, $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$. Continue, “Use a different color to color the new half of the already colored area. Does it represent $\frac{1}{4}$?” (Yes)

3. Then say, “Now let’s model an actual problem by folding two other fractions in this same manner. Let’s say that Laura has $\frac{3}{4}$ of a large candy bar. She decides to give $\frac{1}{2}$ of her candy to her best friend. How much of the *original* whole candy bar will her friend receive? We can solve this problem by using our paper-folding techniques.”
4. “Take a new sheet of paper, which will represent the original whole candy bar. Fold the paper into fourths by folding twice in the same direction, like this:” Model the folding process.



5. “Unfold the paper, color three of the four sections to represent $\frac{3}{4}$, and then refold the paper so that only the colored area is facing you.”
6. “Now fold the paper in half in the opposite direction — a ‘hotdog fold.’ Unfold this fold, and using a new color, shade in $\frac{1}{2}$ of the already colored area. What does this represent?” Answer: $\frac{1}{2}$ of $\frac{3}{4}$ of 1, or $\frac{1}{2}$ of $\frac{3}{4}$
7. “Next, open the paper fully and count the sections of the original that are colored with the last color you used. What fraction of the original is shaded in the new color?” $\left(\frac{3}{8}\right)$ “What can we conclude?” Answer: $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$
8. Ask pairs of students to fold paper to represent the problems shown on the transparency “Paper-Folding Problems.” When they are finished, ask them to demonstrate their solutions to another pair of students. Call for responses and discussion from the whole group once pairs have come to a consensus.
9. *Closing Activity:* Demonstrate the following for the students, saying, “Now let’s apply these skills to a real-life situation. Let’s say you have a candy bar to split with a friend, but you decide that you want to save $\frac{1}{4}$ of it for yourself for tomorrow. So following the same procedure we used with the paper, you start by dividing the whole candy bar in half and then in half again. How many pieces do you have now? (4) You then divide the whole bar in half lengthwise. How many pieces do you have now? (8) Remember, $\frac{1}{4}$ of the whole is to be put aside and saved for tomorrow, so how much are you putting

away for tomorrow (2 pieces or $\frac{2}{8}$ or $\frac{1}{4}$) How much of the whole bar do you have to work with today? (6 pieces or $\frac{6}{8}$ or $\frac{3}{4}$) Applying what we just did with the paper, if you give your friend $\frac{1}{2}$ of today's $\frac{3}{4}$ of the candy bar, how much will that be? (3 pieces or $\frac{3}{8}$, because we originally divided the candy bar into 8 pieces, and $\frac{1}{2}$ of $\frac{3}{4}$ of the whole candy bar is 3 pieces or $\frac{3}{8}$) This leaves how much for you? (5 pieces or $\frac{5}{8}$, i.e., 3 pieces or $\frac{3}{8}$ for today and 2 pieces or $\frac{2}{8}$ or $\frac{1}{4}$ for tomorrow)

Classroom Assessment

Circulate around the classroom to check that each student is folding his/her paper properly. Accurate folding is critical to understanding of the problems and to visualizing the fractions and their multiplication.

Follow-up/Extensions

Working in teams of two, students may use geoboards and geoboard recording paper to find the fractional parts of a garden that will be planted in a variety of flowers. For example, given a garden plot that is 3-by-4 units, the students must allot $\frac{1}{2}$ of the garden to blue flowers and $\frac{1}{2}$ to yellow flowers. In the yellow section, $\frac{1}{2}$ of the $\frac{1}{2}$ will be marigolds, $\frac{1}{6}$ of the $\frac{1}{2}$ will be daisies, and $\frac{1}{3}$ of $\frac{1}{2}$ will be roses. How many units will be used for each kind of flower in the yellow section of the garden? Similarly, in the blue section of the garden, $\frac{2}{3}$ of the $\frac{1}{2}$ will be wildflowers, and $\frac{1}{2}$ of the remaining $\frac{1}{3}$ will be straw flowers. How many units will be used for each kind of flower in the blue section of the garden? Are any units not planted with flowers?

Curriculum Connections

Science: Planting a garden

Mathematics: *Fabulous Fractions*, AIMS Education Publication, <http://www.aimsedu.org>

Literature: Pallotta, Jerry. *The Hershey's Milk Chocolate Fraction Book*, Scholastic, 1999.

Paper-Folding Problems

1. At a certain school, $\frac{3}{4}$ of the teachers are female, and $\frac{1}{3}$ of the female teachers are single. What fraction of the teachers are single females? Use paper folding to determine this.

Complete this statement with a correct fraction: “Taking $\frac{1}{3}$ of the result of taking $\frac{3}{4}$ of something is the same as taking _____ of it.”

2. Michelle has $\frac{2}{3}$ of her monthly allowance left. She plans to save $\frac{1}{4}$ of the money she has left. What fraction of the original monthly allowance does she plan to save?

3. Justin has $\frac{1}{3}$ of a very, very large pizza left. He eats $\frac{3}{4}$ of the remaining pizza while watching a rented video. What fraction of the original pizza did he eat?

4. How did the answers to questions 1 and 3 compare? Is there any difference between $\frac{1}{3}$ of $\frac{3}{4}$ and $\frac{3}{4}$ of $\frac{1}{3}$? Explain.

5. Make up a paper-folding problem to challenge your group.

Percent Grid Patterns

Reporting Category

Number and Number Sense

Overview

Students will color 10-by-10 grids in different patterns to identify and represent equivalent fractions, decimals, and percents.

Related Standards of Learning

4.4, 6.1, 7.1

Approximate Time

45 minutes

Objective

Students will identify equivalent fractions, decimals, and percents.

Prerequisite Understandings/Knowledge/Skills

- Understand that fractions may be expressed as decimals and as percents
- Understand that base-ten blocks and their representations — i.e., 10-by-10 grids — are area/region fraction models that can help clarify the concept of equivalent fractions

Materials Needed

- “Base-Ten Grids” transparencies, with and without shading
- A copy of the handout “Base-Ten Grids,” without shading, for each student
- Colored pencils/crayons
- Overhead markers
- Base-ten 100 flat

Instructional Activity

1. *Initiating Activity:* Discuss how many squares make up a 10-by-10-squares grid. Present the 10-by-10 base-ten 100 flat and the 10-by-10-squares grid. Emphasize that the two are equal. For purposes of this activity, the students are going to use a representation of the base-ten 100 flat — i.e., a 10-by-10-squares grid. This is a representation that they could draw, but a handout has already been created that would save time. Remind the students that one grid represents 1 whole that has been divided into 100 equal-sized parts. (*Note:* Remember that we are again working from the concrete [base-ten flat] to representational [grid] to abstract.)
2. Show the base-ten transparency grid that has been shaded in and ask, “Who has a quick estimate of the percent of the squares that are shaded?” Record responses and ask, “How did you determine how many squares? How did you estimate the percent of the whole that is shaded?”
3. Write below the grid on the transparency the percent and the equivalent fraction and decimal of the shaded portion. Ask volunteers to come up to the overhead and shade in a portion of the three remaining grids. Next, have the volunteers ask the class what percent of each grid is shaded.
4. It is important to write each decimal in “regular” fraction form, “expanded” base-ten fraction form, decimal form, and percent form — for example, $\frac{3}{8} = \frac{37}{100} + \frac{5}{1,000} = .375 = 37.5\%$. Continue to have volunteers participate in shading the transparency in order to produce a variety of fractions, decimals, and percents so that students have ample practice as an entire class before they move into individual work.

5. *Closing Activity*: Distribute colored pencils and a handout “Base-Ten Grids” to each student. Assign each student four *different* percents to color in their four grids and to label appropriately with the percent, the common and expanded base-ten fractions, and the decimal representation. For example, assign 2%, 5%, 7%, 13%, 25%, and 48% and common fractions such as thirds, fourths, fifths, sixths, and eighths. Grids should be displayed in the classroom.

Classroom Assessment

During the activity, observe the students as they complete their grids, and ask them to explain the steps they are taking. Check for accurate expression of fractions, decimals, and percents, both in written form and in conversation. Encourage students to share their explanations with the rest of the class.

Follow-up/Extensions

Promote relative thinking by introducing “Fraction Islands” to familiarize students with another way to “see” the relationships among fractions, decimals, and percents. Students should be encouraged to generate equivalent forms of commonly used fractions, decimals, and percents. These could be displayed in “Poster Proof” format and presented to the class.

Curriculum Connections

Science: ProTeacher Science Activities, <http://www.proteacher.com>
EdHelper Science Activities, <http://www.edhelper.com>

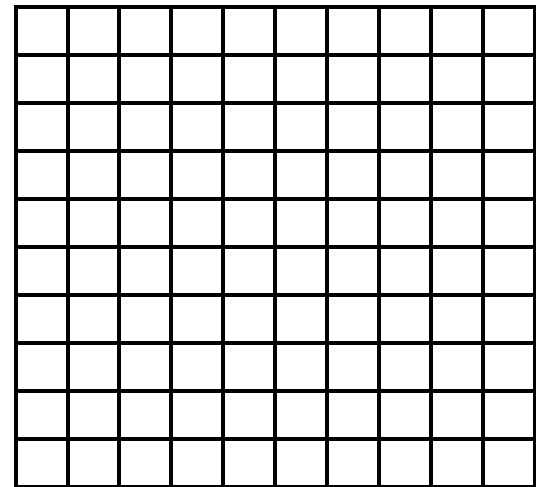
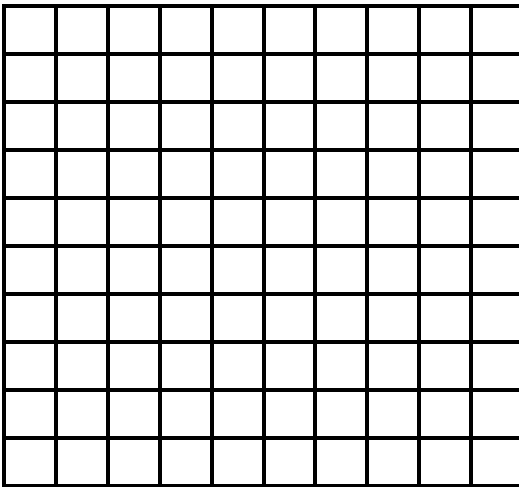
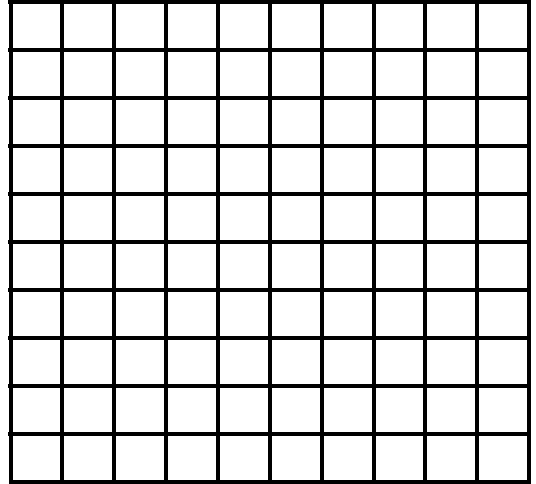
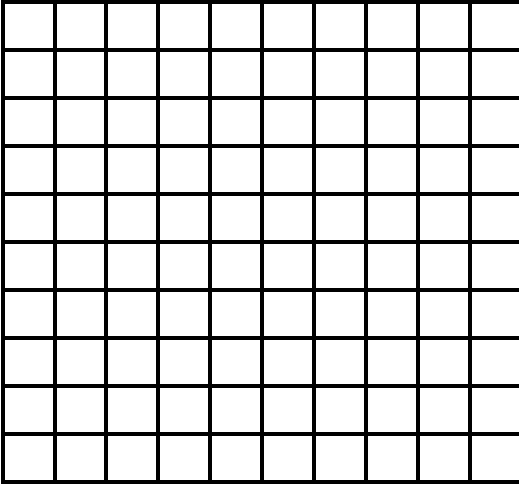
Social Studies: Topographical maps

Mathematics: Activities from “A to Z Math Stuff,” <http://atozmathstuff.com>

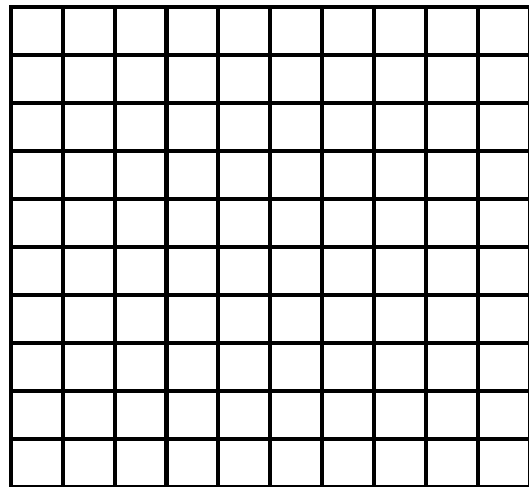
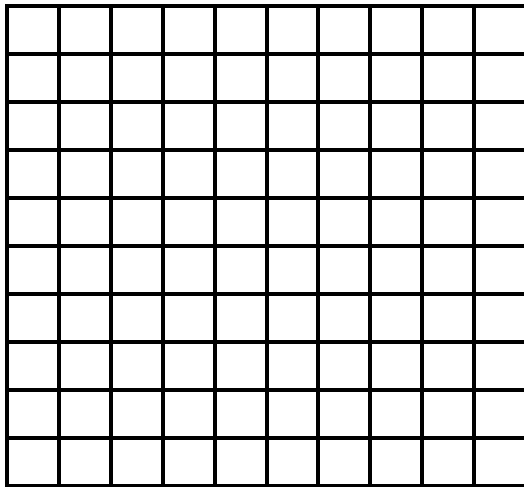
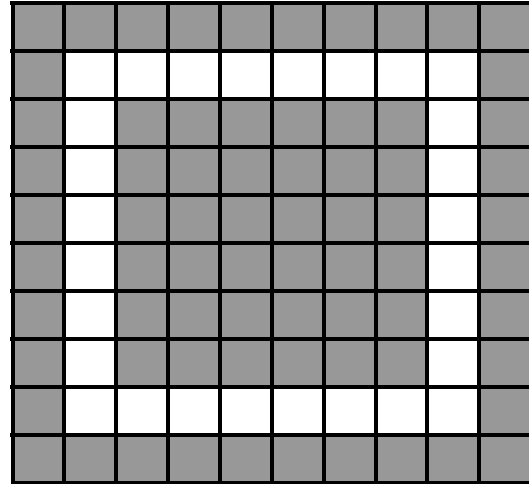
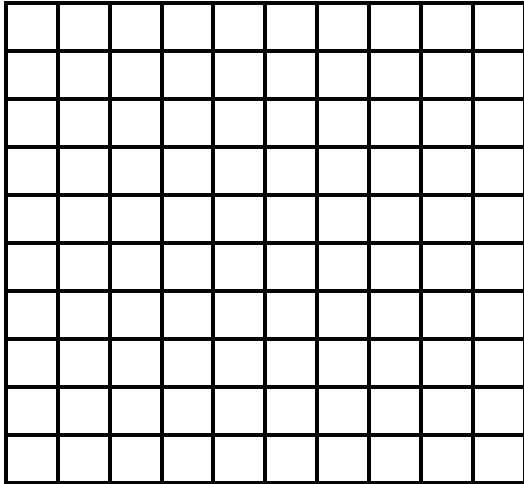
“Visualizing Fractions to Decimals to Percents Using Web-based Manipulatives,”
<http://mason.gmu.edu>

“Counting the Rice,” <http://www.arcytech.org>. This activity is a little complicated, but it is definitely worth examining and perhaps adapting to your class’s needs. Look for the base-ten block activity at this Web site.

Base-Ten Grids



Base-Ten Grids



Playground Problem

Reporting Category	Computation and Estimation
Overview	Students will apply what they have learned about adding and subtracting fractions to a real-life situation.
Related Standards of Learning	6.6a, 6.8, 7.4a, 8.3
Approximate Time	30 minutes

Objectives

- Students will estimate and develop strategies for adding and subtracting fractions.
- Students will gain experience in selecting the appropriate operation to solve problems.

Prerequisite Understandings/Knowledge/Skills

- Understand that a fraction is a way of representing a part of a whole
- Understand that the denominator tells the number of equal-sized parts in a whole
- Understand that the numerator tells the number of equal-sized parts being considered
- Understand how to represent a given part of a whole
- Understand how to add fractions
- Understand how to subtract fractions

Materials Needed

- Individual sets of fraction strips or other fraction manipulatives
- A copy of each of the handouts “Playground Problem,” “Current Playground Map,” and “Future Playground Map” for each group of students
- Transparencies of “Playground Problem” and “Current Playground Map”
- Chart paper
- Markers
- Tape

Instructional Activity

1. Introduce the problem by showing and explaining the transparency “Current Playground Map.” Then show the transparency “Playground Problem,” and review the problem with the students so they will understand what they need to do.
2. Discuss with the class named fractional areas of the playground. Be sure students understand the meaning of “two equal parcels (sections) of land.”
3. Divide the class into four groups for a roundtable. Have the students in each group take turns counting the fractional parts of an area of the current playground and marking it with a fraction. The map should be passed from person to person until every playground area is marked with the appropriate fraction.
4. Have the students work in pairs to solve the problem. At each table, have the pairs discuss their solutions and come to a consensus.
5. Give each group the handout “Future Playground Map” and some markers. Have them draw/record their solution and show how they found it, justifying their answer. Monitor the groups as they work.
6. Tape the “Future Playground Maps” to the wall, and allow the groups to circulate to examine the solutions of the other groups. Allow them to write suggestions or comments on the sheets as they make their examinations.
7. Have the groups return to their tables and discuss with the whole class their observations and comments. Be sure to discuss any discrepancies.

Classroom Assessment

As students work, circulate among the groups, observe the work, and listen to the discussions. Be sure everyone understands the task. Check solutions for completeness as well as accuracy. Be sure each group records how they found the solution or justified their answer.

Playground Problem

The playground at Central Elementary School is formed from two equal parcels (sections) of land, which have been subdivided into 12 assorted areas with specific purposes, as shown on the map. The P.T.A. has agreed to donate money to redesign and update the playground. They want the existing 12 areas combined into four larger areas with the following specifications:

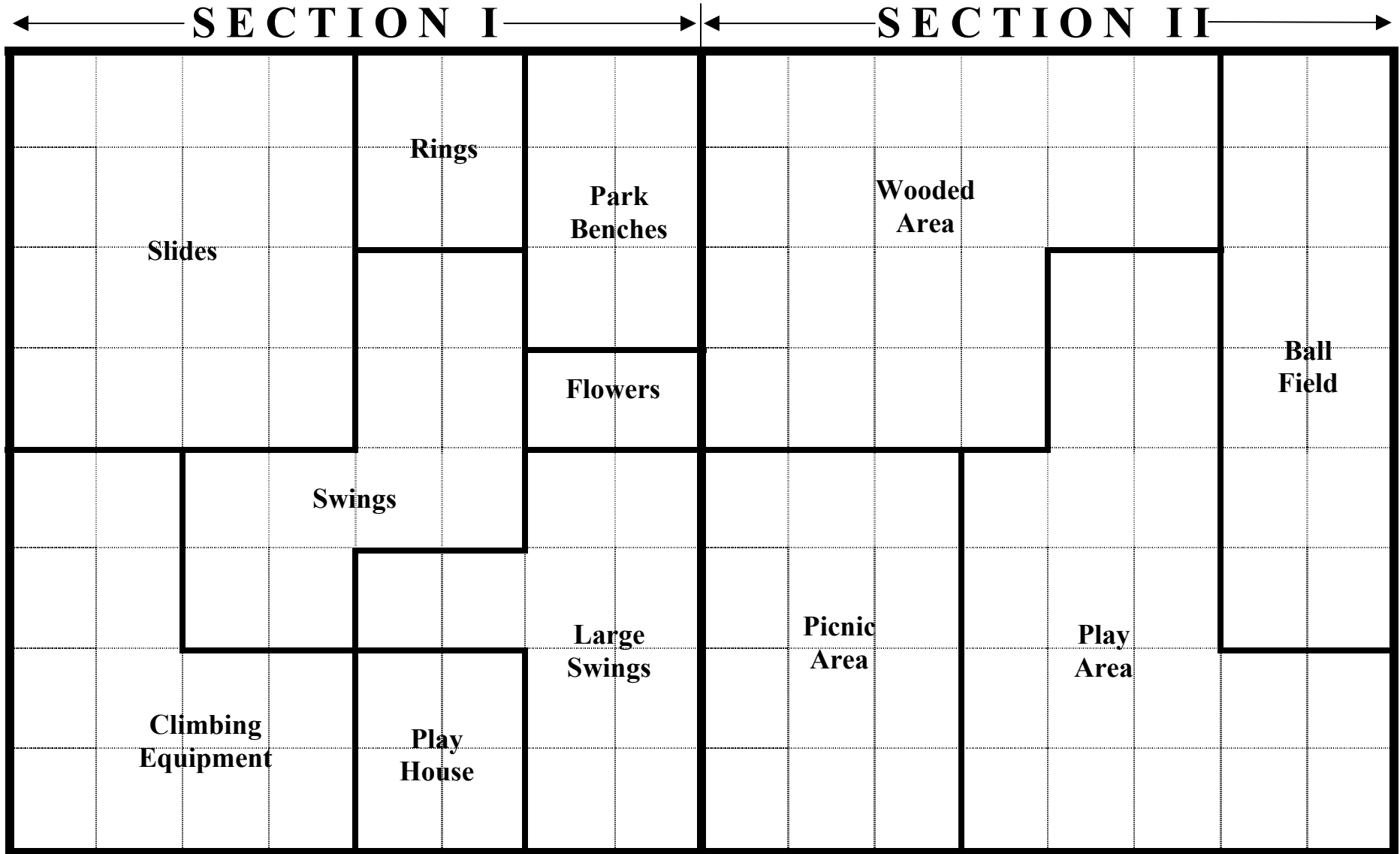
1. After the renovation, all 12 of the existing areas of the playground will be eliminated, and the whole playground will be divided into four new areas.
2. The ball field area will be joined to one other area, and together they will make up $\frac{1}{2}$ of one of the two equal parcels or sections of land. This will be the new “Play Area.”
3. The area containing the Play House will be combined with two other areas to form the new “Primary Playground.” This area will comprise $\frac{13}{32}$ of one of the two sections.
4. The area that holds the slides will be increased to equal $\frac{1}{2}$ of a section. This will be the new “Playground Equipment Area.”
5. The rest of the land in that section will be added to the park benches area. This will be the new “Park & Picnic Area.”
6. You will be able to walk through each of the four new areas of the playground without having to cross another area.

Use the clues above to find out which areas of the original playground were combined to make the four new areas. Explain your answer.

Draw a map of the new playground, and outline each of the four new areas. What fraction of the total playground area will be in each of the four new areas?

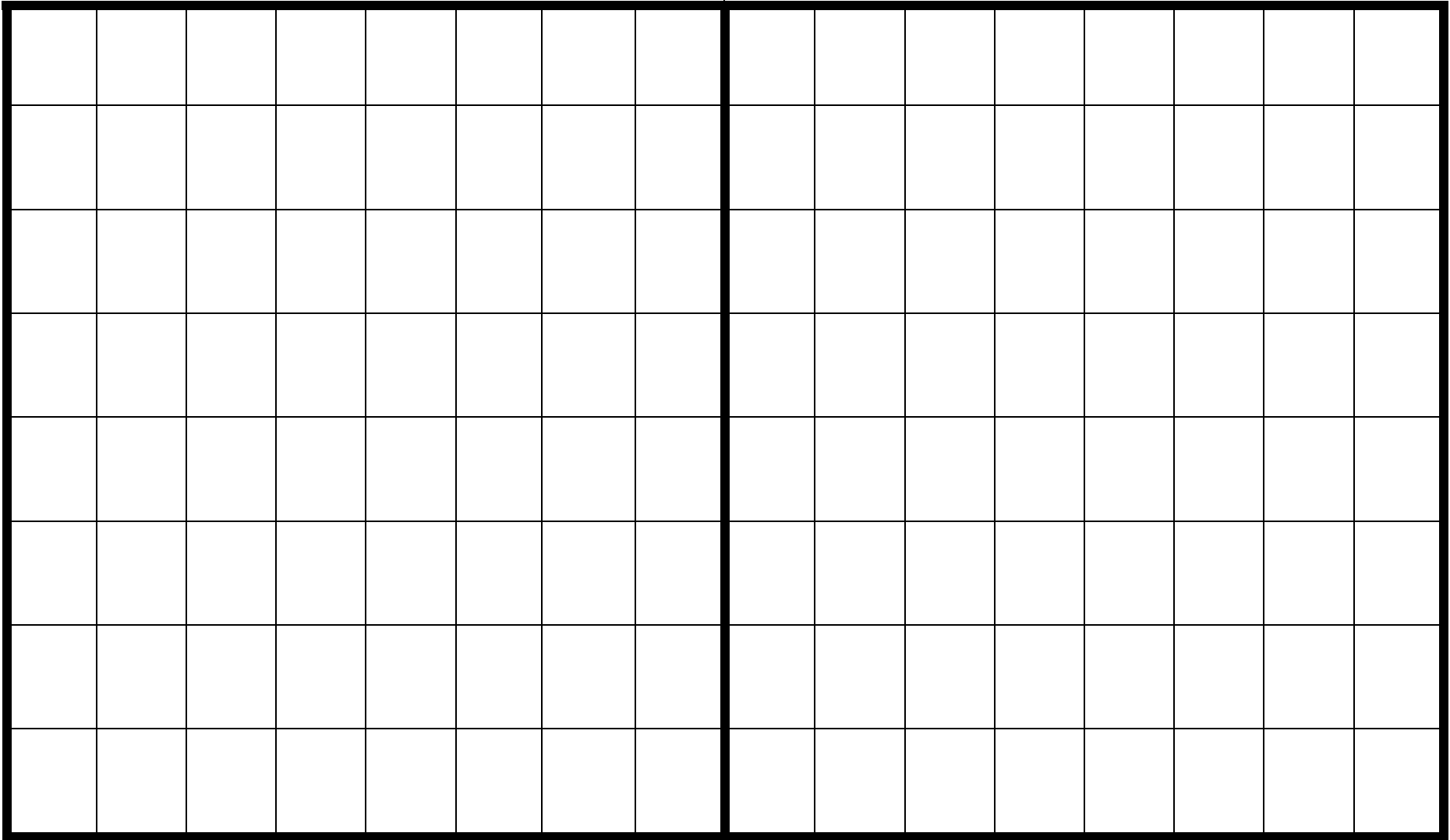
Do you agree with the design of the playground? Explain any changes you would make in the location of equipment or park areas.

Current Playground Map



Future Playground Map

← SECTION I → | ← SECTION II → →



Population Density

Reporting Categories Number and Number Sense; Computation and Estimation; Patterns, Functions, and Algebra

Overview Students will determine a method of calculating population density, using themselves and a designated 2-yd.-by-2-yd. square. Population density will be represented in two ways.

Related Standards of Learning 6.2, 7.6, 8.3, 8.17

Approximate Time 20 minutes

Objectives

- Students will create a unit ratio.
- Students will use proportional reasoning in a practical application.

Prerequisite Understandings/Knowledge/Skills

- Understand how to write fractions, using data
- Understand how to find equivalent fractions
- Understand how to solve proportions

Materials Needed

- Masking tape
- Measuring tape
- Calculators

Instructional Activity

1. *Initiating Activity:* To help the class grasp the concept of population density, conduct the following activity. Have the students use the measuring tape to mark off on the floor a square that is approximately 2-yd.-by 2-yd. Outline the outside square and the grid with masking tape.



2. Have eight students stand within the 2-yd.-by 2-yd square. Discuss as a group possible methods of determining the number of students per square yard. The discussion should move to a “unit ratio” of two people in one square yard. Be certain that the class understands the term *unit ratio*.
3. Ask, “What is the ‘mean number’ of people per square yard?” Make sure the students understand the meaning of the term *mean* and that the mean number indicates the *average* number. Show them that in this case, it is another way of representing the unit ratio, or two people per one square yard.
4. Have the eight students reposition themselves a number of times. Each time ask what is the mean number of people per square yard. The class should come to understand that no matter how the eight students are arranged within the space, there will always be two students per one square yard and that this ratio represents the population density for the taped area.
5. *Closing Activity:* Extend the problem to 15 people. What is now the mean number of people in one square yard? Be sure that the class understands why there are 3.75 people per one square yard.

Classroom Assessment

Make sure the concepts of *mean* and *unit ratio* are understood. Check student answers for the problem of 10 people in the square to be sure the concept application was extended to a different number.

Curriculum Connections

Science: Population disturbances and factors that increase or decrease population size and density
Social Studies: Census taking, population studies

The Scoop on Ice Cream

Reporting Categories

Computation and Estimation; Patterns, Functions, and Algebra

Overview

Students will use data involving favorite ice cream flavors to predict the amount of each flavor that would be needed to serve a crowd of people.

Related Standards of Learning

7.4, 7.6, 8.3, 8.17

Approximate Time

15 minutes

Objectives

- Students will use data to predict an outcome, using proportional reasoning.
- Students will use unit proportions and equivalencies for finding solutions.

Prerequisite Understandings/Knowledge/Skills

- Understand how to write proportions
- Understand how to solve proportions
- Understand how to calculate equivalent fractions

Materials Needed

- A copy of the handout “Scoop-on-Ice-Cream Recording Sheet” for each student
- Calculators

Instructional Activity

1. *Initiating Activity:* Give each student a copy of the handout “Scoop-on-Ice-Cream Recording Sheet,” and discuss the information about favorite ice cream flavors. Ask, “What percent of the people prefer chocolate ice cream?” “Vanilla ice cream?” “Strawberry?”
2. Review with the class the important measurement equivalents that will help them solve the problem.
3. Read the Scrumptious Scoops problem with the class.
4. Have the class work in pairs to solve the problems concerning amounts of ice cream.
5. *Closing Activity:* When students have completed the problems, share their solutions and strategies for solving.

Classroom Assessment

Pay particular attention to the conversions of units and the proportions used to find answers. Make sure students recognize amounts that “make sense” when their answers are calculated.

Solution

1. How many people will prefer chocolate ice cream?
 - a. If 29% of the people prefer chocolate, then 29% of 650 people = $(.29)(650) = 188.5$ or 189.
Hence, 189 people prefer chocolate.
or
 - b. If 29% of the people prefer chocolate, then $\frac{29}{100} = \frac{x}{650}$; therefore, $x = \frac{(650)(29)}{100} = 188.5$ or 189.
Hence, 189 people prefer chocolate.

2. How many half-gallons of chocolate ice cream will be needed?
- If $\frac{1}{2}$ cup is needed for each person, then $\frac{.5 \text{ cups}}{1 \text{ person}} = \frac{x \text{ cups}}{189 \text{ people}}$; therefore, $x = (189)(.5) = 94.5$.
Hence, 94.5 cups of chocolate ice cream are needed for 189 people.
 - If one half-gallon equals 8 cups, then $\frac{1 \text{ half-gallon}}{8 \text{ cups}} = \frac{x \text{ half-gallons}}{94.5 \text{ cups}}$; therefore, $x = \frac{94.5}{8} = 11.81$ or 12. Hence, 12 half-gallons of chocolate will be needed.
3. If everyone is served a scoop, how many half-gallons of ice cream will be served? How many pounds will that be?
- If one scoop of ice cream equals $\frac{1}{2}$ cup, then the number of cups of ice cream needed for 650 people is $(.5)(650) = 325$ cups. If one half-gallon equals 8 cups, then the number of half-gallons of ice cream needed for 650 people is $325 \div 8 = 40.6$ or 41 half-gallons of ice cream.
 - If one gallon weighs 5 pounds, then one half-gallon weighs $5 \div 2 = 2.5$ pounds. Therefore, 40.6 half-gallons weigh 101.5 or 102 pounds of ice cream.

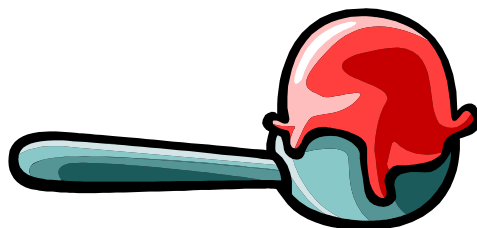
Follow-up/Extensions

Do the “The Ice Cream Recipe” activity found on the handout, either as an individual activity or as a whole-class activity.

Curriculum Connections

Social Studies: Economics, goods and services

Scoop-on-Ice-Cream Recording Sheet



The Three Favorite Flavors of Ice Cream*

Favorite Flavor	Percent of Those Polled
Vanilla.....	55%
Chocolate	29%
Strawberry.....	16%

Scrumptious Scoops is a very popular ice cream parlor in a small town in Virginia. To celebrate the Fourth of July, the store decided to serve free single scoops of its three most popular flavors to the audience at the Independence Day outdoor band concert. Mr. Scrumptious decided that he could determine how much ice cream he would need by using the data provided by the International Ice Cream Association. The town estimated that approximately 650 people would attend the band concert.

1. Assuming everyone will want a free scoop of ice cream, how many people would you expect to prefer chocolate?
2. How many half-gallons of chocolate ice cream should Mr. Scrumptious plan to have on hand to give to those people?
3. If the representatives from Scrumptious Scoops serve everyone at the band concert a scoop of ice cream, how many half-gallons of ice cream will they serve? How many pounds will that be?

Important Measurement Equivalents

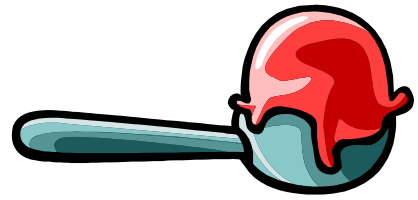
A gallon of ice cream weighs about 5 pounds and contains 4 quarts.

One scoop of ice cream is $\frac{1}{2}$ cup or about 68 grams.

One gallon contains 16 cups, so one half-gallon contains 8 cups.

*International Ice Cream Association Data

Ice Cream Recipe



Make ice cream in plastic zip-lock bags as follows:

1. Combine 3 tablespoons of sugar, a few drops of vanilla extract, and 1 cup of milk in a 1-quart zip-lock bag, and seal the bag tightly. You may add cookie pieces or well-drained fruit to your ice cream mixture if you wish.
2. Put about 2 cups of ice and $\frac{1}{2}$ cup of rock salt in a 1-gallon zip-lock bag. Use small ice cubes, or break the ice into small pieces.
3. Put the smaller bag into the larger bag, and seal it tightly. Then shake the large bag until the ice cream mixture freezes. This step takes some time.

Why do you think this method works for making ice cream?

Using the recipe above, how many students could make ice cream using 8 cups of sugar and 4 gallons of milk? To solve the problem you need to know: 1 cup = 16 tbsp. and 1 gallon = 16 cups. Show your calculations in the box below:

Would there be any ingredients left over? If so, how much? Show your thinking below:

Snowy Egrets

Reporting Categories

Number and Number Sense; Computation and Estimation;
Patterns, Functions, and Algebra

Overview

Students will use the results of a capture-tag-recapture experiment along with proportional reasoning to determine the number of snowy egrets in a population. There is no simulation for this experiment; students will apply the methodology of the deer simulation used in the “Animals Count” activity.

Related Standards of Learning 6.2, 6.8, 7.6, 8.3, 8.17

Approximate Time 15 minutes

Objective

Students will use techniques they have learned for reasoning about ratios and proportions to estimate the size of a population when they cannot count the entire population. In this problem, there will be a direct application of proportional reasoning using available data.

Prerequisite Understandings/Knowledge/Skills

- Understand the “Animals Count” activity
- Understand how to create proportions
- Understand how to solve proportions
- Understand the meaning of a ratio
- Understand how to represent data with fractions

Materials Needed

A copy of the handout “Snowy Egrets Recording Sheet” for each student

Instructional Activity

1. *Initiating Activity:* Hand out the “Snowy Egrets Recording Sheet” and review the background information with the class. Answer questions as they arise.
2. Ask the students to use the methodology from the deer capture-tag-recapture experiment in the “Animals Count” activity to solve the problem. Make sure they show the proportion they use and explain the strategy for their estimate.

$$\frac{2 \text{ tagged egrets}}{50 \text{ egrets}} = \frac{20 \text{ tagged egrets}}{x \text{ egrets}}$$

3. *Closing Activity:* Discuss the estimates with the class. Ask for volunteers to explain why they are confident that their answer is correct. Discuss with the class *why* proportional reasoning works for this kind of problem.

Classroom Assessment

Students should be able use the data from the problem to form the proportion. One method of solving this proportion is to solve the equation for x .

$$x = \frac{(20)(50)}{2} = 500 \text{ egrets}$$

Another method is to realize that 20 is 10 times 2, therefore x would be 10 times 50, or 500 egrets.

Follow-up/Extensions

Additional problems using this method could be assigned for students struggling with forming their proportions. For instance, a problem involving fish in a lake or one involving beavers in a wooded area would be good examples of real-world situations that require similar reasoning.

Curriculum Connections

Science: Population disturbances and factors that increase or decrease population size

Social Studies: Census taking, population studies

Snowy Egrets Recording Sheet



The U.S. Department of Natural Resources manages the bird population in the Chesapeake Bay area on the eastern shore of Maryland. The Department has been tracking the snowy egret population and has discovered that snowy egrets tend to migrate to the same geographical area each year during the warmer spring and summer months. The Department is worried that urban sprawl may be affecting the egret population adversely.

For the past several summers, a biologist studied the snowy egret, a white shore bird that frequents the eastern shore of Maryland near St. Michaels during the summers. Two summers ago, she trapped 20 snowy egrets, tagged, and released them. This past summer, she trapped 50 snowy egrets and found that two of them were tagged. She used this information to estimate the total snowy egret population on the eastern shore of Maryland.

1. Using the biologist's findings, estimate the number of snowy egrets on the eastern shore of Maryland. Explain how you made your estimate.

2. How confident are you that your estimate is accurate? Explain your answer.

Something's Fishy

Reporting Category

Number and Number Sense

Overview

Students will use an area/region fraction model to find and record equivalent fractions in the context of a game.

Related Standards of Learning

4.2, 4.3

Approximate Time

45 minutes

Prerequisite Understandings/Knowledge/Skills

- Understand that equivalent fractions are ways of describing the same amount, using different-sized fractional parts
- Understand that pattern blocks are an area/region fraction model that can clarify the concept of equivalent fractions

Materials Needed

- Pattern blocks (except for the square and the rhombus) or, alternatively, paper pattern blocks and scissors for each pair of students
- A spinner marked with pictures of the pattern blocks for each pair of students
- A paper clip and pencil for each pair of students
- A copy of the handout “Something’s Fishy Rules of the Game” for each pair of students
- A copy of the handout “Something’s Fishy Game Board” for each student

Note: For easier management, duplicate the paper pattern blocks on the following colors of construction paper: hexagons – yellow, parallelograms – blue, trapezoids – red, triangles – green. Game boards and spinners will last longer if they are duplicated on tagboard or some other type of heavy paper and laminated. If paper pattern blocks are used, have the students store them in plastic storage bags when finished.

Instructional Activity

1. *Initiating Activity:* Brainstorm with the students about how it is possible to share a pizza on one day with three friends and the next day to share a pizza with seven friends but still eat the same amount of pizza that you ate the day before. Use the following example to illustrate: “Mike and three friends stopped at a pizza parlor Saturday night and shared a large pizza equally. What does this mean? Yes, they each had the same-size piece, or *equivalent* piece, of pizza. The next day Mike and seven friends stopped at the same pizza parlor for a snack. This time the eight friends shared a large pizza equally among them and then ordered a second large pizza and shared it equally. Did Mike eat more pizza on Saturday or on Sunday? Or did he eat the same amount each day?”
2. Encourage students to draw pizzas to illustrate what Mike and his friends ate each day. Remember: **concrete to representational to abstract**. To insure that students fully understand the situation, you may want to give each student three equal-sized circles and a pair of scissors and walk them through the process of dividing the first circle into four equal-sized parts and then dividing each of the next two circles into eight equal-sized parts. Compare Mike’s part of Pizza One to what he ate from Pizzas Two and Three. Voila! They are equal — i.e., **equivalent fractions**.
3. Now, extend the concept by having the class play “Something’s Fishy.” Explain that the class will be divided into pairs of students and the players in each pair will compete against each other. Give each

pair some pattern blocks (or the paper and scissors alternatives), a copy of the game rules, a spinner with paper clip and pencil, and two game boards. Be sure to demonstrate how to use the spinner if students are not familiar with it. Explain that the object of the game is to cover the game board with pattern blocks completely but without overlaps. The first player to do this is the winner.

4. Explain that for every turn, each player spins the spinner and makes one of three choices: 1) take the pattern block indicated and place it anywhere it fits on his/her board; 2) take other blocks that when fitted together are equivalent to the block indicated and place these anywhere they will fit on the board (They do not have to be placed together but can be placed anywhere separately.); or 3) pass and do nothing. *Important:* Once blocks have been selected and placed, they may not be moved.

Example: A player spins a hexagon but decides to take two triangles and two parallelograms to place on his/her board instead. These four blocks may be placed anywhere on the board; they do not have to be placed together.

5. After the players have played one round of the game, ask them to record equivalent relationships among the blocks. They should consider the hexagon as one “whole” for this purpose.

Example: A player spins a hexagon but decides to take six triangles instead. The player places the blocks on his/her game board and also records this action:

1 hexagon “whole” = 6 triangles; therefore, 1 triangle represents $\frac{1}{6}$ of the whole.

6. After the players have played at least two rounds of the game, discuss the experience. Ask the students to show and describe what they learned about fractional equivalents between and among the pattern blocks. Participants should have noticed that it takes three parallelograms to equal one hexagon; therefore, one parallelogram is equal to one-third of the hexagon “whole.” Two parallelograms equal two-thirds. Two triangles, or two-sixths of the “whole,” are equal to one parallelogram or one third of the whole.

3 parallelograms = 1 hexagon

1 parallelogram = $\frac{1}{3}$ of a hexagon

2 parallelograms = $\frac{2}{3}$ of a hexagon

2 triangles = 1 parallelogram

2 triangles = $\frac{2}{6}$ or $\frac{1}{3}$ of a hexagon

2 trapezoids = 1 hexagon

1 trapezoid = $\frac{1}{2}$ of a hexagon

7. Have the students continue play until each pair has a winner.
8. *Class Management:* After play, have the students collect all the pattern blocks and return them to the appropriate containers. Have students store paper pattern blocks in plastic storage bags for future use.

Game boards and spinners along with paper clips and pencils should be collectively stored. This will facilitate the use of the game as either a math center, a small group activity, or another class activity.

9. *Closing Activity:* Refer to the story at the beginning of activity about Mike and his friends and their visits to the pizza parlor. Encourage students to explain how Mike was able to eat the same amount of pizza on Sunday that he ate on Saturday, even though he was with a different number of friends each day. Have a student demonstrate the solution to the class, using the circle cutouts from the beginning of the activity and the known amounts of pizza that Mike ate. Have the student show how the amounts are equivalent. Then ask a student to explain the equivalent fractions represented by the pattern blocks used in “Something’s Fishy.” How do “Something’s Fishy” and Mike’s story compare? (Both activities contain equivalent fractions, and different combinations may be used to cover the same area.)

Classroom Assessment

Observe the students as they play the game, and check for understanding of equivalent values as they choose the pattern blocks. Ask students to show you the variety of ways that they can cover the area of a hexagon when using only trapezoids, triangles, and parallelograms. They should be able to see that it is possible to cover the area of a hexagon with triangles only or trapezoids only. Likewise, they should be able to explain the combinations that can be used, for example, one trapezoid, one parallelogram, and one triangle. Encourage them to show a variety of combinations.

Follow-up/Extensions

Use “Covering the Whole Unit” and “Cover-up” as opportunities for students to both extend their learning and demonstrate an understanding of equivalent fractions. “Something’s Fishy” is an excellent small group activity for those students who complete their work early in class and need an opportunity to either practice equivalent fractions or extend their understanding of fractions.

Curriculum Connections

- Science: ProTeacher Science Activities, <http://www.proteacher.com>
EdHelper Science Activities, <http://www.edhelper.com>
- Social Studies: Native Americans and their products
Maritime flags
- Mathematics: Pallotta, Jerry. *The Hershey’s Milk Chocolate Fraction Book*, Scholastic, 1999.
Activities from “A to Z Math Stuff,” <http://atozmathstuff.com>

Something's Fishy Rules of the Game

This is a game for two players. Each player uses his/her own game board, and the players together use a spinner and a set of pattern blocks.

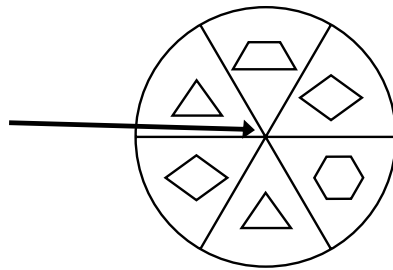
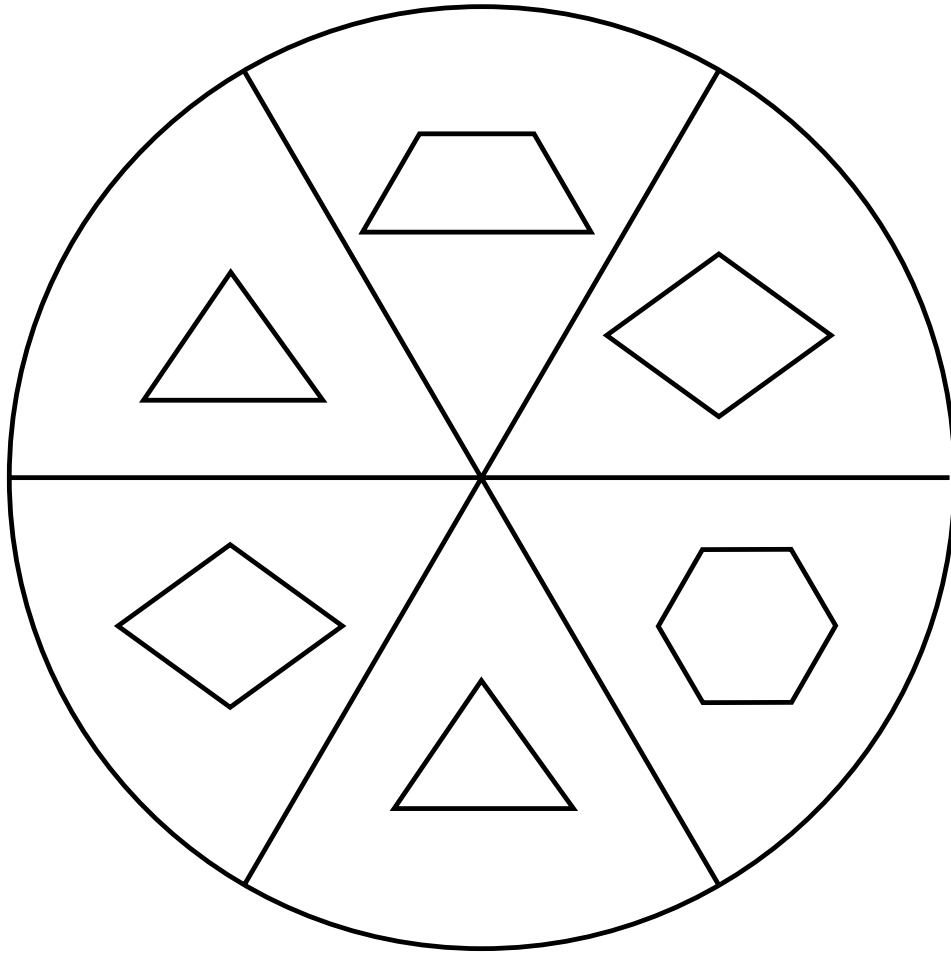
OBJECT OF THE GAME

To be the first player to cover your game board with pattern blocks completely but without overlaps

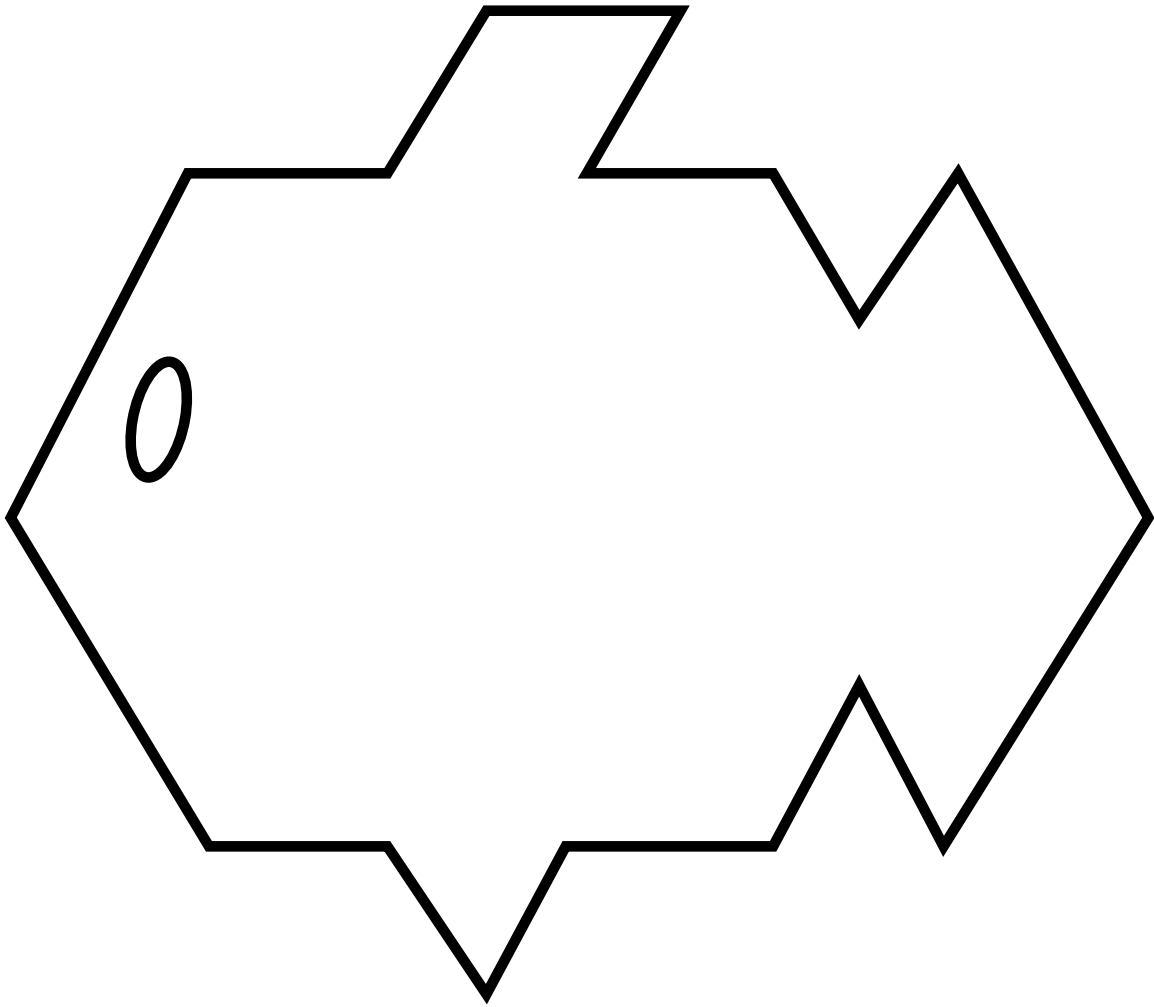
1. Take turns spinning the spinner. Use a pencil and paper clip to activate the spinner.
2. After each spin, make one of these three choices:
 - a. Take the pattern block indicated on the spinner, and place it anywhere it fits on your game board.
 - b. Take other blocks that when fitted together are *equivalent* to the block indicated, and place these anywhere they fit on your game board. They do not have to be placed together.
 - c. Pass and do nothing.
3. Once blocks have been selected and placed on your board, they may not be moved.
4. The first player to cover his/her game board completely but without overlaps is the winner.

Be ready to talk about strategies you found useful while playing “Something’s Fishy.”

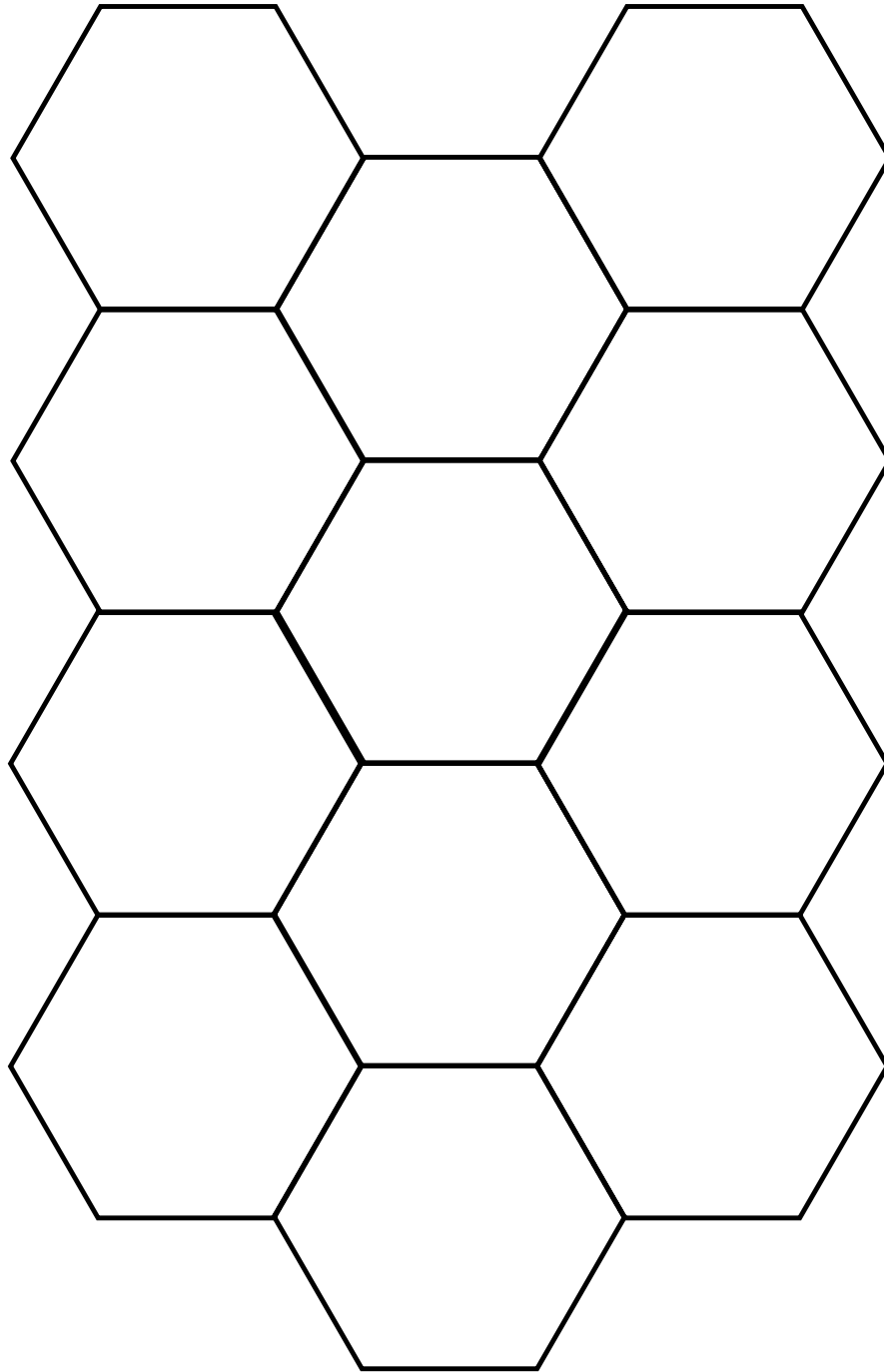
Something's Fishy Spinner



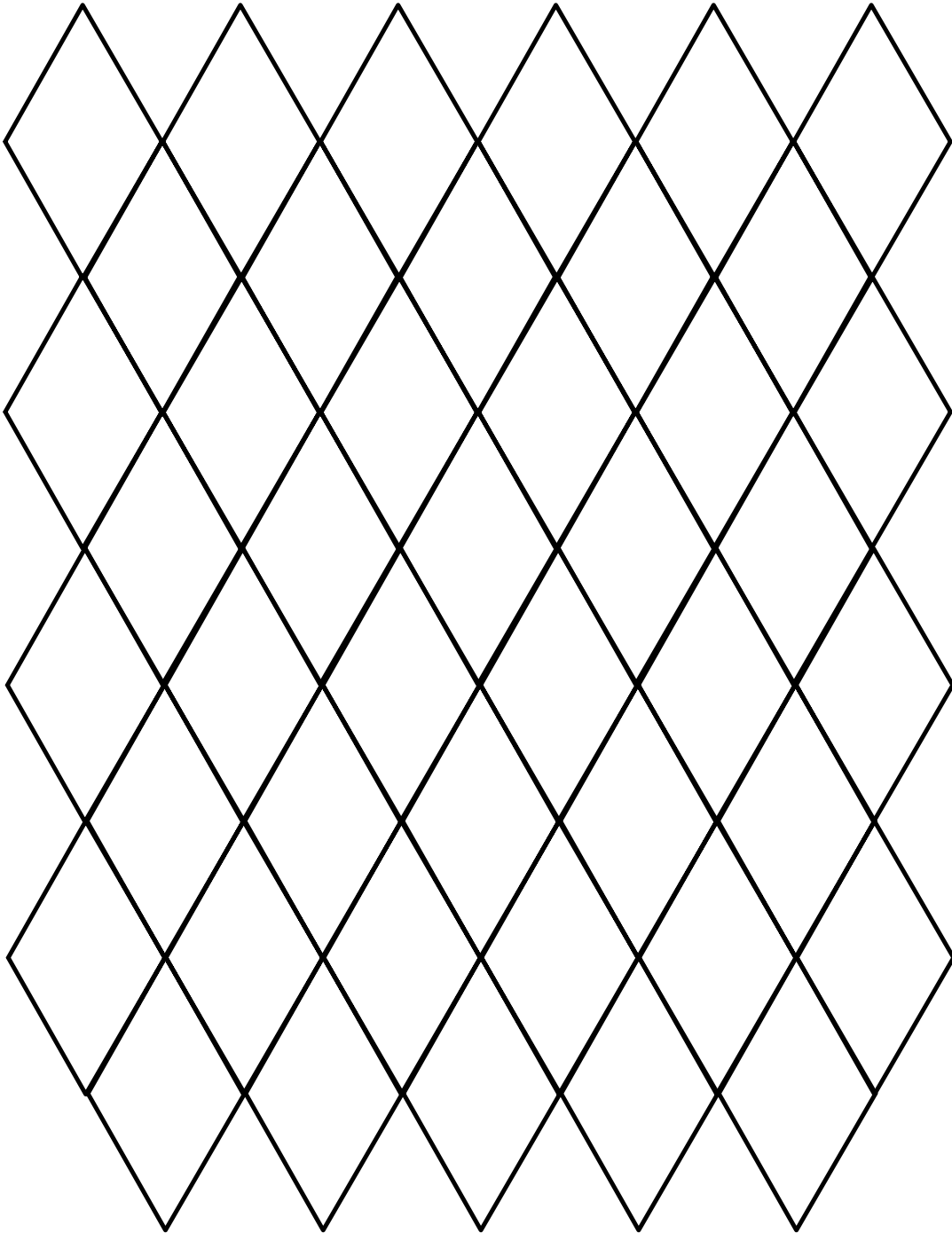
Something's Fishy Game Board



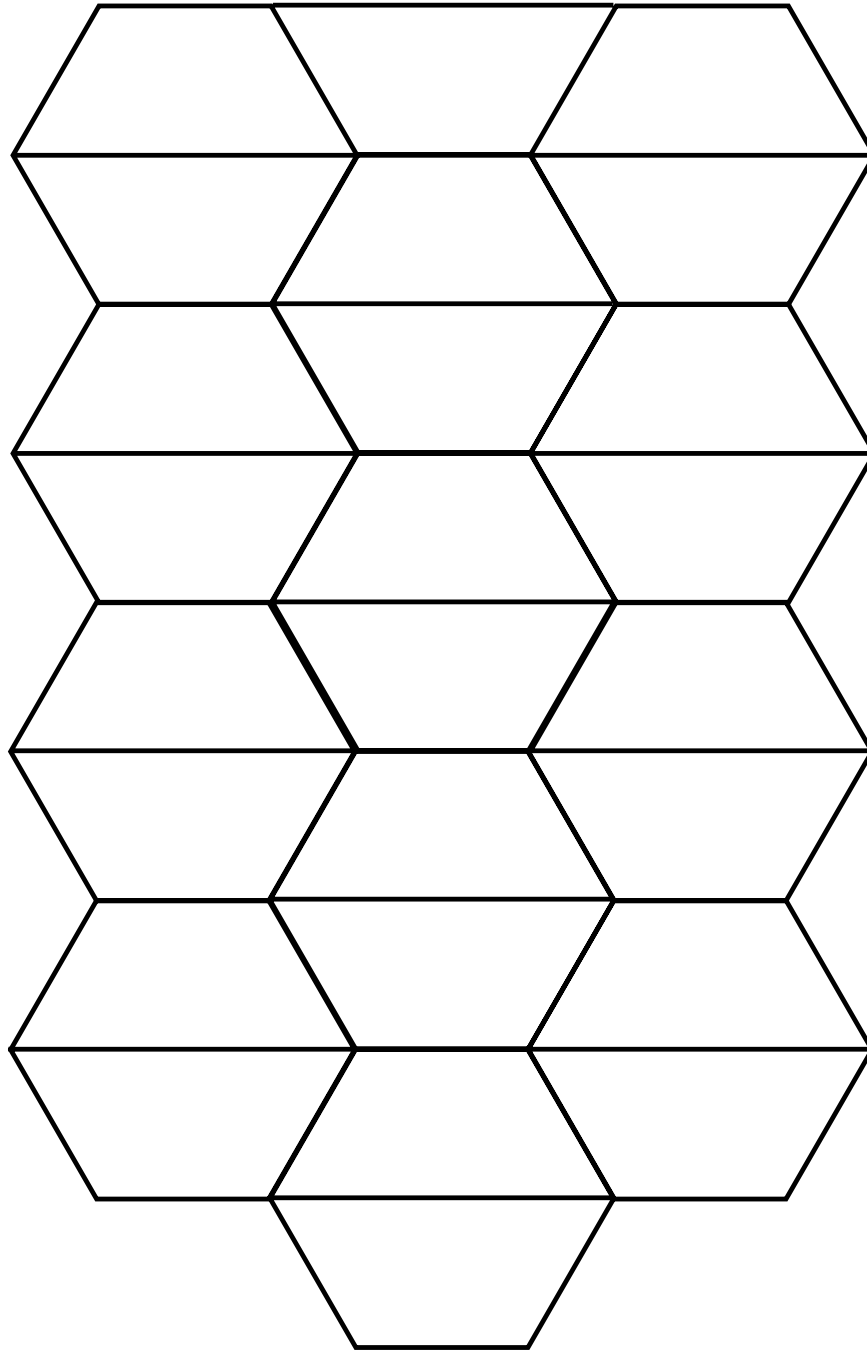
Something's Fishy Hexagon Pattern Blocks



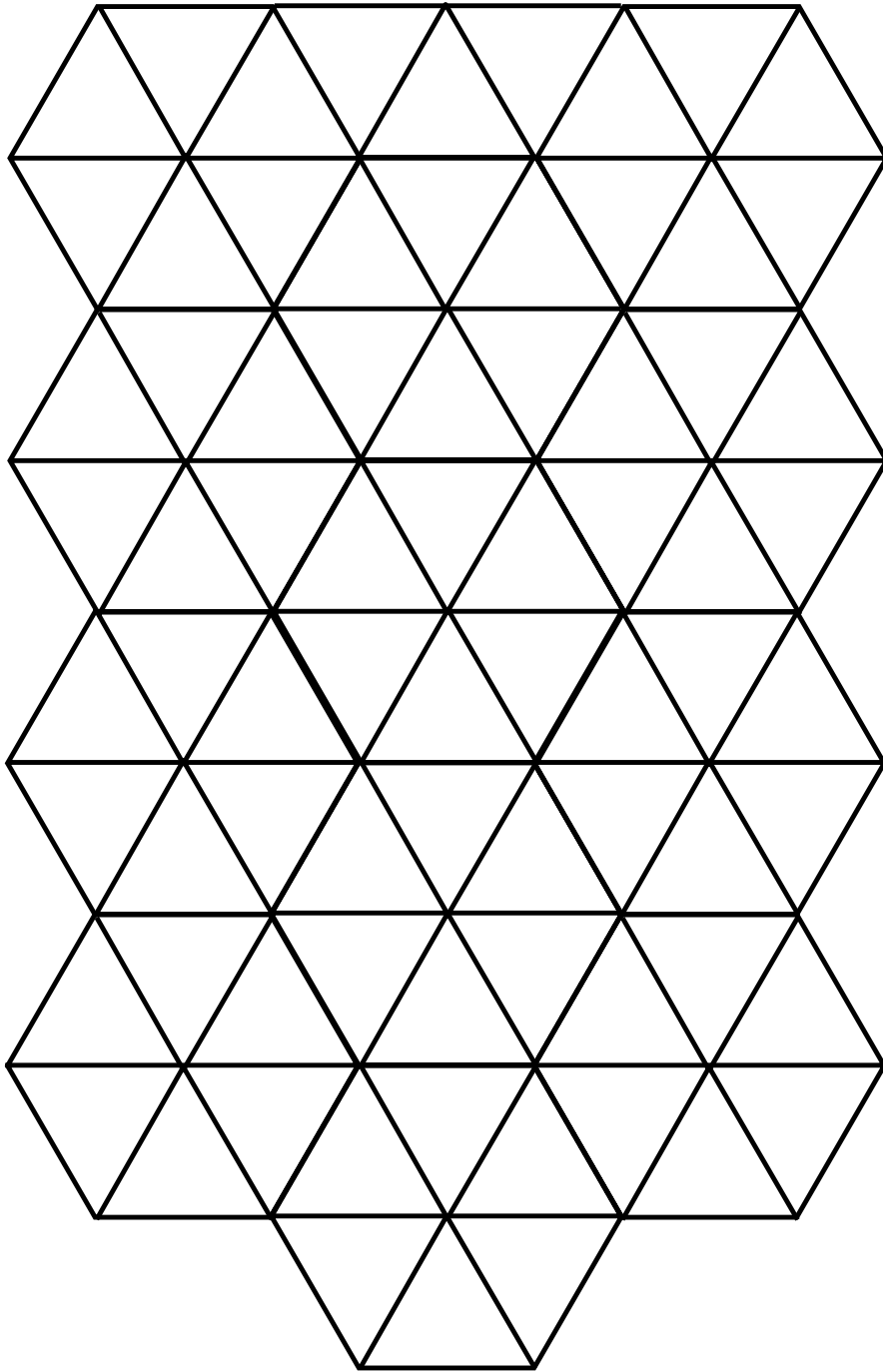
Something's Fishy Parallelogram Pattern Blocks



Something's Fishy Trapezoid Pattern Blocks



Something's Fishy Triangle Pattern Blocks



Spin to Win

Reporting Category Overview

Number and Number Sense
Students will model, record, and compare fractional amounts in the context of a game. This activity provides an opportunity to work with the concepts and procedures involved in comparing and ordering fraction.

Related Standards of Learning 4.2a, 4.3, 6.4

Approximate Time 20 minutes

Objectives

- Students will create and compare fractional amounts within the context of a game.
- Students will connect fractional notation with concrete models.

Prerequisite Understandings/Knowledge/Skills

- Understand that a fraction is a way of representing part of a unit whole
- Understand that models can be used to judge the sizes of fractions
- Understand benchmarks as an aid in comparing the sizes of fractions

Materials Needed

- Transparency of the “Spin-to-Win Game Board”
- Overhead markers
- A “Spin-to-Win Spinner” for each students
- A paper clip and pencil for each student
- An assortment of fraction manipulatives for each student

Instructional Activity

1. Demonstrate how to play the game by playing a round with the whole class. Tell them that you will be Player A and will go first. Use the transparency of the game board so that all can see.
2. Step #1: For round one, spin the spinner four times. After each spin, write the number in either one of the boxes or on one of the lines below “Player A.” Once written, a number cannot be moved. After the fourth spin, there will be a fraction (formed by the numbers inside the boxes) and two “rejected” numbers (those on the lines).
3. Have the students (Player B) take their turn. Spin the spinner four more times, and ask the students to tell you where to place the numbers on the transparency game board — either inside the boxes or on the lines under “Player B.”
4. Step #2: Ask students: “How could I show this fraction?” and use their input to model the fraction you have made with commercial or handmade manipulatives and with a drawing. Ask for a volunteer to model the fraction for Player B with commercial or handmade manipulatives and with a drawing.
5. Step #3: As a group, compare both players’ fractions and write a fraction “sentence” to show the comparison, for example, $\frac{3}{4} > \frac{2}{5}$, or $\frac{3}{6} = \frac{6}{12}$. Use the models to check each other’s ideas and settle any dispute. The player with the larger fraction wins one point; if the two fractions are equivalent, each player wins a point. Each game lasts four rounds.

6. Group the students in pairs to play “Spin to Win.”
7. *Closing Activity:* After students have had a chance to play the game, discuss their experiences. Ask them to describe any strategies they may have used that worked particularly well.

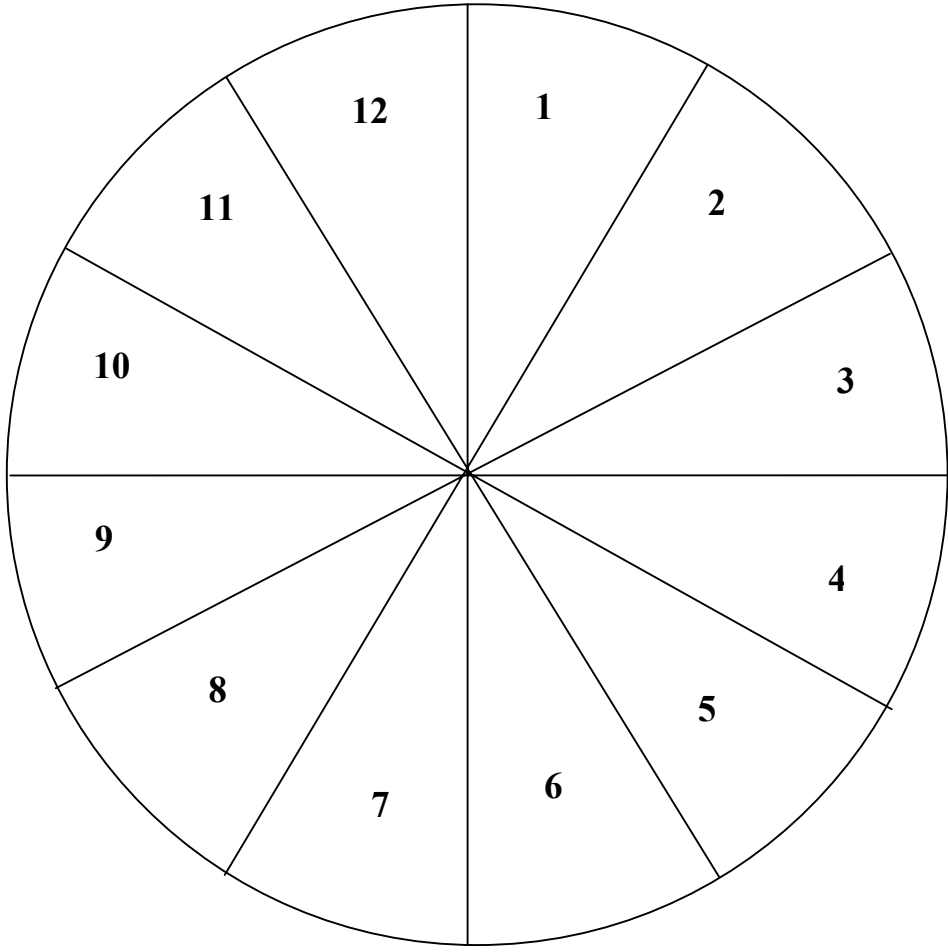
Classroom Assessment

While the students are playing, circulate and listen to the discussions that take place. Watch for the student who does not seem to understand the concept of getting a “large” fraction.

Follow-up/Extensions

Players could spin the spinner only four times and both players use the same numbers to fill the game board each round. Players could be required not only to provide a model, but to write a descriptive situation to match the fractions they create.

Spin-to-Win Spinner



Spin-to-Win Game Board

ROUND ONE

Step	Player A	Player B
#1	$\frac{\square}{\square} \text{ — } \text{ —}$	$\frac{\square}{\square} \text{ — } \text{ —}$
#2	Model:	Model:
#3	Fraction sentence: _____	Fraction sentence: _____
	Score: _____	Score: _____

ROUND TWO

Step	Player A	Player B
#1	$\frac{\square}{\square} \text{ — } \text{ —}$	$\frac{\square}{\square} \text{ — } \text{ —}$
#2	Model:	Model:
#3	Fraction sentence: _____	Fraction sentence: _____
	Score: _____	Score: _____

ROUND THREE

Step	Player A	Player B
#1	$\frac{\square}{\square} \text{ — } \text{ —}$	$\frac{\square}{\square} \text{ — } \text{ —}$
#2	Model:	Model:
#3	Fraction sentence: _____	Fraction sentence: _____
	Score: _____	Score: _____

ROUND FOUR

Step	Player A	Player B
#1	$\frac{\square}{\square} \text{ — } \text{ —}$	$\frac{\square}{\square} \text{ — } \text{ —}$
#2	Model:	Model:
#3	Fraction sentence: _____	Fraction sentence: _____
	Score: _____	Score: _____

The In-between Game

Reporting Category

Number and Number Sense

Overview

Students will model decimals on grid paper and compare decimals in the context of a game.

Related Standards of Learning 4.2, 4.4, 5.1, 6.4, 7.1

Approximate Time

30 minutes

Objectives

- Students will use 10-by-10 grids to explore relationships between decimals.
- Students will demonstrate correct ordering of decimals by choosing a number that falls between two decimals.

Prerequisite Understandings/Knowledge/Skills

- Understand the part-whole concept of fractions
- Understand the decimal place values and how to read and write a decimal

Materials Needed

- Blank 10-by-10 grids for each student
- A copy of the handout “In-between Game” for each pair of students
- Overhead transparency of the “In-between Game” handout
- Overhead markers

Instructional Activity

1. *Initiating Activity:* Hand out multiple copies of 10-by-10 grids to each pair of students. Model on the overhead how to divide the grid into 10 equal-sized parts and shade one part. Write this as $\frac{1}{10}$ or 0.1. Ask each pair to similarly shade $\frac{2}{10}$ or 0.2 on their first grid. Next ask them to shade $\frac{20}{100}$ or 0.20 on another grid. Have them compare the two shaded decimals. Ask, “Which shaded decimal is the largest?” (They are the same or equivalent.)
2. Have the students use two more grids to shade the decimals 0.3 and 0.34. Ask which is larger (0.34), and call for responses and reasons. Name the corresponding fractions ($\frac{3}{10}$ and $\frac{34}{100}$). Use two more grids to shade in 0.46 and 0.5. Ask which is larger (0.5), and call for explanations. Make sure that the students understand how to compare decimals by place value, and clear up any misconceptions.
3. Give each pair a copy of the handout “In-between Game.” Explain the rules of the game. (*Note:* you may need to model a game first on the overhead to get the students started, answering questions as you go.)
 - a. The first player chooses a decimal number and writes it on the first row in the first column. The second player chooses a second decimal different from the first (smaller or larger) and writes it on the first row in the third column directly across from the first number. Example:

0.5 _____ 0.7

- b. The first player then chooses a decimal number that is “in between” the original two numbers, records this number on the second row in the middle column, and crosses out the smallest (first) number. Example:

0.5	<u> </u>	<u>0.7</u>
<u> </u>	<u>0.68</u>	<u> </u>

- c. The first player now writes the two remaining numbers on the third row in the first and third columns, and the game continues. Example:

0.5	<u> </u>	<u>0.7</u>
<u> </u>	<u>0.68</u>	<u> </u>
<u>0.68</u>	<u> </u>	<u>0.7</u>

- d. The second player now chooses a number “in between” these numbers, and so on for five to ten rounds. Example:

0.5	<u> </u>	<u>0.7</u>
<u> </u>	<u>0.68</u>	<u> </u>
0.68	<u> </u>	<u>0.7</u>
<u> </u>	<u>0.689</u>	<u> </u>
<u>0.689</u>	<u> </u>	<u>0.7</u>

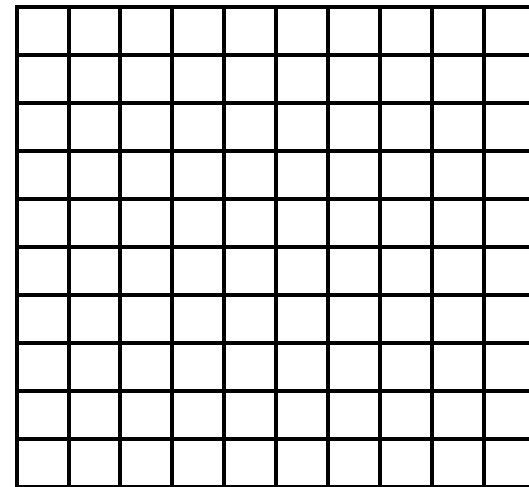
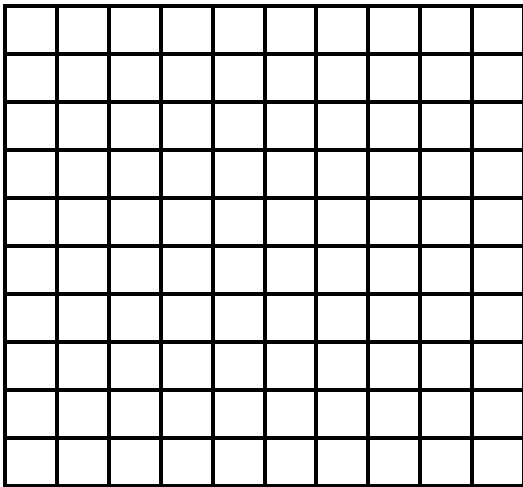
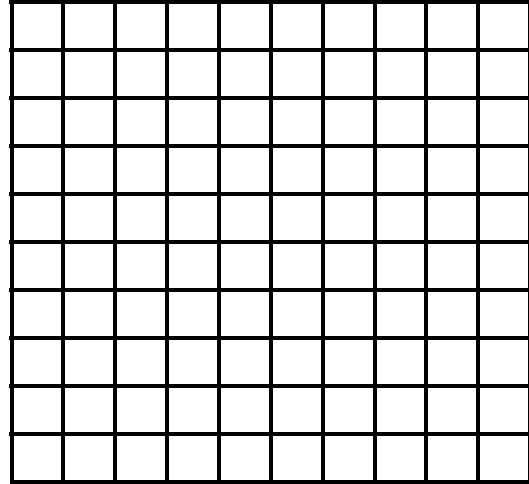
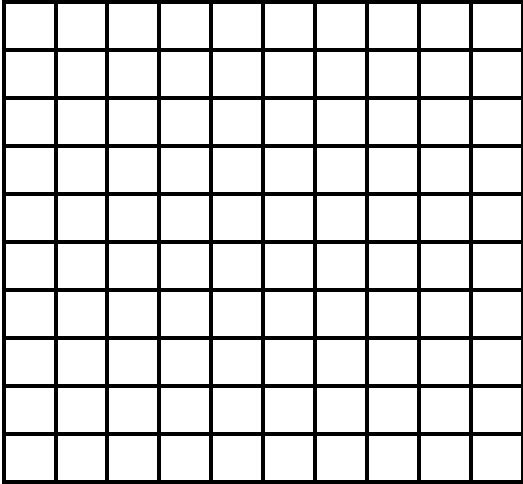
4. After the students have had the opportunity to play the game on their own, ask several of the pairs to display their games on the overhead for all to see and to share their game plays and strategies.

Follow-up/Extensions

As an alternative, this game could be played using fractions, percents, or a combination of fractions, decimals, and percents.

The In-between Game

Base-Ten Grids



Virginia Population Density

Reporting Categories Number and Number Sense; Computation and Estimation; Patterns, Functions, and Algebra

Overview Given the population numbers of Virginia and West Virginia and the number of square miles of land in each state, students will determine population density for each state and then decide which state has the greater population density. Based on this information, students will develop a strategy that will help them determine how many citizens of one state would have to move to the other state to make the population densities equal.

Related Standards of Learning 6.2, 7.6, 8.3, 8.17

Approximate Time 30 minutes

Objective

Students will work with population densities and unit proportions to understand the relationship of population to available area as a population density.

Prerequisite Understandings/Knowledge/Skills

- Understand how to create and solve proportions
- Understand how to use a calculator
- Understand how to find a mean or average

Materials Needed

- Calculators
- A copy of the handout “Virginia Population Density Recording Sheet” for each student

Note: It is helpful for the students to first complete the “Population Density” activity in order to understand the concept of unit proportions. Refer back to the concept of mean or average population.

Instructional Activity

1. *Initiating Activity:* Give the students copies of the “Virginia Population Density Recording Sheet” and the following data taken from the 1990 Census:

	Virginia	West Virginia
Population	6,187,000 people	1,793,000 people
Land Area	40,767 sq. mi.	24,231 sq. mi.

2. Ask the students to develop a strategy to determine the population density of each state and then calculate the population densities.
3. Instruct the students to determine which state has the greater population density and then to determine how many citizens of one state would have to move to the other state to create equal *population densities* (not equal *populations*). Remind the students that they should be prepared to describe the strategy used and to explain their reasoning.
4. *Closing Activity:* When the students are finished, have them explain their strategies for solution to the class. Their solutions should include information about how many citizens would have to move from one state to the other. Answers should include all calculations.

Classroom Assessment

Monitor the class as the students work on and develop the following solutions. If incorrect strategies are conceived, note where the students lost the right track and lead them back to the proper process by asking questions. Solutions are as follows:

- To determine the population densities of the two states, make sure the students set up the proper ratios and solve them correctly:

Virginia: $\frac{6,187,000 \text{ people}}{40,767 \text{ sq. mi.}} = \frac{x \text{ people}}{1 \text{ sq. mi.}}$; therefore, $x = 151.76$ people per square mile, which is the population density of Virginia.

West Virginia: $\frac{1,793,000 \text{ people}}{24,231 \text{ sq. mi.}} = \frac{x \text{ people}}{1 \text{ sq. mi.}}$; therefore, $x = 74.00$ people per square mile, which is the population density of West Virginia.

Now the students can see that Virginia has a higher population density than West Virginia.

- An alternative method of representation is the following:

Virginia: $\frac{40,767 \text{ sq. mi.}}{6,187,000 \text{ people}} = .007$ square miles per person in Virginia

West Virginia: $\frac{24,231 \text{ sq. mi.}}{1,793,000 \text{ people}} = .013$ square miles per person in West Virginia

While this method does not yield population density, it does show that with less area of land per person, Virginia is more densely populated than West Virginia.

It is plain that Virginia citizens would have to move to West Virginia to equalize the population densities. There is more than one way to find out how many people would have to move. A possible solution using proportional reasoning is the following:

- Find the total area in both states. (64,998 square miles)
- Find the total population of both states. (7,980,000 people)
- Find the ratio of the total population of both states to the total area of both states; this ratio represents the mean or average population density that needs to be achieved if the population densities are to be equalized. ($\frac{7,980,000 \text{ people}}{64,998 \text{ sq. mi.}} = \frac{x \text{ people}}{1 \text{ sq. mi.}}$; therefore, $x = 122.77$ people per square mile)
- Subtract this average population density from the population density of Virginia. ($151.76 - 122.77 = 28.99$ people per square mile) This tells the number of Virginia citizens per square mile who would have to move to West Virginia.
- Multiply this number of Virginia citizens per square mile who would have to move times the total number of square miles in Virginia ($28.99 \times 40,767 = 1,181,835$ people). This is the number of Virginia citizens who would have to move to West Virginia to equalize the population densities.

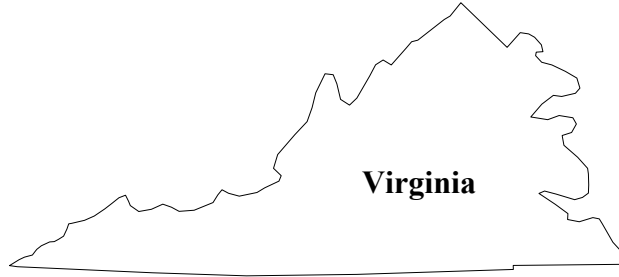
Follow-up/Extensions

Have the students confirm or prove their solutions to the problem of how many Virginia citizens would have to move. Have them subtract this number of people (1,181,835 people) from the total population of Virginia and then add this same number of people to the total population of West Virginia, thereby creating the census after the move takes place. Then have them use these new population numbers to refigure the population densities of both states. They should quickly discover that the new population densities are the same — 122.77 people per square mile.

Virginia Population Density



Population: 1,793,000 people
Area: 24,231 sq. mi.



Population: 6,187,000 people
Area: 40,767 sq. mi.

1. Which state, West Virginia or Virginia, has the greater population density?
Show your work.

2. How many citizens of one state would have to move to the other state to create equal *population densities*? Be prepared to tell the strategy you used and explain your reasoning.

Waste Paper

Reporting Categories

Computation and Estimation; Patterns, Functions, and Algebra

Overview

Students will use proportional reasoning to determine the number of cubic yards of landfill space that can be saved by recycling the Sunday newspapers.

Related Standards of Learning

7.6, 8.3, 8.17

Approximate Time

20 minutes

Objective

Students will use unit ratios and formulate proportions to solve contextual problems about recycling and environmental issues.

Prerequisite Understandings/Knowledge/Skills

- Understand how to create and solve proportions
- Understand how to use a calculator

Materials Needed

- A copy of the handout “Waste Paper Recording Sheet” for each student
- Calculators

Note: Reviewing volume concepts (cubic measurements) with the students might help them solve this problem more efficiently.

Instructional Activity

1. *Initiating Activity:* Hand out a “Waste Paper Recording Sheet” to each student, and read the recycling example #1 with the class.
2. Have students work in pairs to develop a strategy to solve the problem. Once they complete their work, have them share their strategies for solution and compare their answers.
3. *Closing Activity:* Continue to example #2 and read it with the class. Have the students solve the problem and explain their strategies.

Classroom Assessment

Monitor the class as the pairs of students work on and develop the following solutions. If incorrect strategies are conceived, note where the students lost the right track and lead them back to the proper process by asking questions.

Example #1 Solution: If 100% of the newspapers were recycled, then producing the Sunday papers with recycled paper would save 500,000 trees. Dividing 500,000 by 17 equals 29,412 tons of recycled paper that would be saved. Multiplying 29,412 by 3.3 equals 97,060 cubic yards of landfill that would be saved.

Example #2a Solution: In a year, a family of four would produce $(2.7 \text{ lbs.})(4 \text{ people})(365 \text{ days}) = 3,942 \text{ lbs.}$ of garbage. Dividing 3,942 by 50 equals 78.84 cubic feet of garbage. Dividing 78.84 by 27 equals 2.92 cubic yards of garbage.

Follow-up/Extensions

Using the given information, have the students solve the following problem: Each year the United States generates about 450 million cubic yards of solid waste. Mrs. Robinson’s classroom is 42 ft. long by 30 ft. wide by 12 ft. high. How many rooms of this size would be needed to hold the garbage generated by the United States in one year?

Solution: Mrs. Robinson's Room is 15,120 cubic feet (42 ft. \times 30 ft. \times 12 ft.), which is 560 cubic yards (15,120 \div 3³). 450,000,000 cubic yards per year divided by 560 cubic yards in the classroom equals 803,571 classrooms this size.

Curriculum Connections

Literature: Clement, Rod. *Counting on Frank*, Gareth Stevens Publishing, 1991.

Which Is Closer?

Reporting Category Overview

Number and Number Sense
Students will use common fractions, decimals, and benchmarks in a game context to estimate the sum of chosen fractions and determine if the sum is closer to 0, 1, or 2.

Related Standards of Learning 4.2, 4.3, 4.9, 5.7

Approximate Time 20 minutes

Objectives

- Students will estimate fraction and decimal sums.
- Students will compare decimal and fraction sums to the benchmarks 0, 1, and 2.

Prerequisite Understandings/Knowledge/Skills

- Understand how to add common fractions, using fraction strips or other manipulatives.
- Understand how to add decimals.

Materials Needed

- A copy of the handout “Which Is Closer? Game Cards — Fractions” for each group of students
- A copy of the handout “Which Is Closer? Game Cards — Decimals” for each group of students
- A copy of the handout “Which Is Closer? Sum Cards” for each student
- Scissors
- Fraction strips or hundredths strips and hundredths grids
- Calculators

Note: This game should follow introductory work with fraction strips or other manipulatives. By the time the students undertake this activity, they should have a working knowledge of common fractions, such as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$, and how to represent them as parts of a whole unit.

Instructional Activity

1. *Initiating Activity:* Pose a sample question, such as “Is sum of $\frac{1}{4}$ and $\frac{7}{8}$ greater than or less than 1?” Ask the students to estimate the answer. Once they have determined that the sum is greater than 1, ask if it is closer to 1 or to $1\frac{1}{2}$. Do not introduce rules or algorithms at this point. Instead, encourage students to use fraction manipulatives and common benchmarks or drawings to explore and reason.
2. Divide the students into groups of two to four players. Give each player a “Which Is Closer? Sum Cards” handout and each group a “Which Is Closer? Game Cards — Fractions” handouts. Ask the students to cut out all their cards. Assign one player per group the task of calculating the sums for the game.
3. Instruct all players to hold their personal Sum Cards in their hands and to place the group Game Cards face down in a pile in the center of the table. Model the following rules of the game for one round to make sure the students understand the play:
 - a. The first player draws two cards from the Game Cards pile and turns them up in the center of the table. Each player estimates the sum of the two fractions shown on the Game Cards (using

fraction strips if necessary), and decides if the sum is closer to 0, 1, or 2. Each student then places the corresponding Sum Card (0, 1, or 2) face up on the table.

- b. After each player has placed his/her Sum Card on the table, the assigned player uses the calculator or some other method to determine the sum.
 - c. The first player who put the correct Sum Card down collects the two Game Cards. If there is a tie, each player gets one Game Card. (If necessary, draw cards from the pile.)
 - d. Each player who put down the wrong Sum Card must return one of his/her Game Cards to the bottom of the pile if he/she has any Game Cards.
 - e. Play continues with the winner of the round turning over two more Game Cards from the pile.
 - f. When all the Game Cards have been used, or there is only one left, the game ends. The winner is the player with the most Game Cards.
4. *Closing Activity:* Give each player a chance to describe the estimation strategies that worked best. You may choose to call on each group to describe one of its strategies.

Follow-up/Extensions

Have the students play the game with the decimal Game Cards.

Which Is Closer? Game Cards — Fractions

$\frac{1}{5}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{2}{5}$
$\frac{3}{8}$	$\frac{1}{8}$	$\frac{3}{10}$	$\frac{1}{2}$	$\frac{7}{8}$
$\frac{7}{10}$	$\frac{1}{10}$	$\frac{3}{8}$	$\frac{9}{10}$	$\frac{3}{2}$
$\frac{5}{3}$	1	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{8}$
$\frac{4}{3}$	$\frac{7}{5}$	$\frac{3}{5}$	$\frac{5}{4}$	$\frac{7}{4}$

Which Is Closer? Game Cards — Decimals

0.5	1.5	2	0.25	0.75
1.25	1.75	0.33	0.67	1.33
0.1	0.2	0.3	0.4	0.6
0.7	0.8	0.9	1.1	1.2
1.8	1.9	0.125	0.375	0.625

Which Is Closer? Sum Cards

0	1	2
0	1	2
0	1	2
0	1	2
0	1	2
0	1	2
0	1	2

Which Is More?

Reporting Category

Number and Number Sense

Overview

Students will compare the values of fractions, using manipulatives and drawings and will provide explanations and generalizations to support their comparisons. No formal algorithms will be used.

Related Standards of Learning

4.3

Approximate Time

30 minutes

Objective

Students will develop fraction number sense by comparing fractions and explaining why one fraction is greater than the other.

Prerequisite Understandings/Knowledge/Skills

- Understand that the denominator tells the number of equal-sized parts in a whole
- Understand that the numerator tells the number of equal-sized parts being considered
- Understand that the value of a fraction is dependent on both the denominator and the numerator

Materials Needed

- A variety of manipulatives (e.g., fraction strips or circles, counters)
- A copy of the handout “Which Is More?” for each student
- Overhead or regular markers
- Blank transparencies or chart paper

Instructional Activity

1. Discuss with students the importance of developing fraction number sense by using conceptual understanding of fractions rather than procedures, such as cross multiplication or finding common denominators, in order to compare the sizes of two fractions.
2. Distribute the handout “Which Is More?” Have the students work in pairs to complete the handout without using cross multiplication or finding common denominators. Encourage students to use different manipulatives to model the problems and drawings to record their work. Make sure students write down their own rules or generalizations that can be used to compare subsets of different types of fractions. (*Important:* These “rules” should be developed by the students in class discussion and not just given to them as rules to follow to solve a problem.) Possible rules or generalizations:
 - a. **Same-Size Parts:** If the denominators are the same (if you’re comparing same-size parts), the fraction with the larger numerator is the greater fraction. For example, $\frac{5}{7} > \frac{4}{7}$.
 - b. **Same Number of Parts:** If the numerators are the same (if you’re comparing the same number of parts), the fraction with the smaller denominator is the greater fraction. For example, $\frac{3}{5} > \frac{3}{7}$.
 - c. **Reference or Benchmark Point:** When the numerators and denominators are different, the fractions can be compared to a common benchmark or reference point such as “one-half” of “one

whole.” For example, $\frac{2}{5} < \frac{6}{8}$ because $\frac{2}{5}$ is less than half the number of fifths needed to make a whole and $\frac{6}{8}$ is more than half the number of eighths needed to make a whole.

- d. **Distance from a Benchmark:** When the numerators and denominators are different, one might consider the size of the fractional part or parts needed to complete “one-half” or “one whole.” For example, $\frac{11}{12} > \frac{4}{5}$ because each fraction is one fractional part away from one whole, and twelfths are smaller than fifths.
4. Have several pairs of students use transparencies or chart paper to record their rules or generalizations and share them with the whole class. As students share their ideas, spend considerable time discussing responses to the problems, using manipulative models and drawings.

Which Is More?

There are several candy bars, cookies, and pizzas for you and your friends to share. However, in each case you must choose which fractional part you want. Work with your partner to find out which fraction is greater. You may not use the procedures of cross multiplication or finding common denominators. To complete your task, use different manipulatives to model the problem and make drawings to record your work. Circle the fraction that is greater:

1. $\frac{1}{3}$ or $\frac{1}{4}$

2. $\frac{3}{4}$ or $\frac{3}{8}$

3. $\frac{2}{5}$ or $\frac{1}{5}$

4. $\frac{2}{3}$ or $\frac{1}{4}$

What rules or generalizations have you discovered that can help you decide which fraction is greater?

- _____
- _____
- _____

As you continue, write down how you decided which fraction is greater. You may use the rules you created to help you but no formal procedures or algorithms.

5. $\frac{4}{5}$ or $\frac{13}{12}$ _____

6. $\frac{5}{6}$ or $\frac{9}{10}$ _____

7. $\frac{17}{8}$ or $\frac{17}{10}$ _____

8. $\frac{1}{3}$ or $\frac{3}{12}$ _____

9. $\frac{6}{4}$ or $\frac{11}{8}$ _____

10. $\frac{4}{3}$ or $\frac{14}{12}$ _____

When you are done, check over your explanations. Can you find any exceptions to your rules? You may revise your rules and write down any new rules.

Who Has 100 Things?

Reporting Category Overview

Number and Number Sense

Students will create a fraction number line and use it to find equivalent fractions, decimals, and percents. Students will develop decimal number sense by forming different subsets of 100 objects. Students will also make connections among different representations (fractions, decimals, and percents) for the same amount.

Related Standards of Learning 4.2, 5.2, 6.1, 7.1

Approximate Time 45 minutes

Objectives

- Students will find fractional parts and determine the equivalent decimal and percent.
- Students will develop decimal number sense and make connections among different representations (fractions, decimals, and percents) for the same amount.

Prerequisite Understandings/Knowledge/Skills

- Understand that a fraction is a way of representing part of a unit whole
- Understand that a decimal is a way of representing part of a unit whole
- Understand that a percent is a way of representing part of a unit whole

Materials Needed

- Rolls of adding machine tape
- One pre-cut meter strip of adding machine tape for each student
- Markers
- Scissors
- 100 similar objects packaged for each small group
- A copy of the handout “Table of 100 Things” for each small group
- A transparency of the handout “Table of 100 Things” for each small group
- Yarn, string, or jump ropes; three pieces of string two-to-three feet long
- Chart paper
- Protractors (optional)
- A copy of the handout “Mosaic Art” for each student for use in the extension activity

Instructional Activity

Part I

1. *Initiating Activity:* Have each student estimate a length of adding machine tape that is one meter long and cut that estimate from a roll of tape. Have each student decorate his/her “meter” strip in some manner that he/she will be able to recognize. After the strips have been decorated, have the students tape their strips to a tape stripe on the wall. Strips should be taped from the shortest to the longest in bar-graph fashion. Measure the strips with a meter stick to determine which one is closest to one meter long. Now is a good time to talk about ways to estimate a meter: a meter is a little bit longer than a yard; it is about the distance from the floor to a doorknob; or it is the about the width of a twin bed.
2. After a brief discussion about a meter, distribute a pre-cut meter strip of tape to each student. Have the students place their strip horizontally on the desk or table in front of them.
3. Have the students write “0” on the left end of the strip and “1” on the right end of the strip. Discuss briefly that the strip now represents one unit. Model the labeling as you go.

4. Have the students fold the right end of their strip over to the left end and crease. Have them open their strip and observe that the crease makes it appear to be divided into two equal parts. Have the students write " $\frac{0}{2}$ " under the 0 on the left end, " $\frac{1}{2}$ " on the crease, and " $\frac{2}{2}$ " under the 1 on the right end. Model this as you go. After this step, discuss briefly with the students that they now have two ways to write the quantity zero — 0 and $\frac{0}{2}$ — and two ways to write the quantity one — 1 and $\frac{2}{2}$.
5. Have students fold the strip in half again and then fold it a second time and crease. Have them open their strip and observe that it appears to be divided into four equal parts.
6. Have the students write " $\frac{0}{4}$ " under the 0 and $\frac{0}{2}$, " $\frac{1}{4}$ " at the first crease, " $\frac{2}{4}$ " under the $\frac{1}{2}$, " $\frac{3}{4}$ " at the third crease, and " $\frac{4}{4}$ " under the 1 and $\frac{2}{2}$. After this step, discuss briefly with the students that they now have three ways to write the quantity zero — 0, $\frac{0}{2}$, and $\frac{0}{4}$. Point out that they now have two ways to express the quantity one-half — $\frac{1}{2}$ and $\frac{2}{4}$ — and three ways to express the quantity one — 1, $\frac{2}{2}$, and $\frac{4}{4}$.
7. If you wish, the same procedure can be done for eighths. Have the students fold their strip in half, in half again, and in half a third time. The strip will now look like it has been divided into eight parts. Have them follow the same procedure for labeling. After labeling, point out equivalent fractions.
8. Folding into thirds is a bit tricky. Have the students think of the strip as a belt and lap the two ends toward the each other, adjusting until the folded lengths are of equal length. Model this folding for them. Have them crease and then label the three parts as they have done before. Be sure to point out that no thirds line up with halves, fourths, or eighths.
9. Have the students turn their strip over to the blank side, making sure that the 0 end is still to the left and the 1 end is to the right. Have them label the left end 0 and the right end 100. Now is the time to discuss the fact that one meter is equivalent to 100 centimeters and that is why 100 was chosen for the label.
10. Have the students fold their strip into two parts, open it, and label the parts. Have them write " $\frac{0}{100}$ " under the 0 on the left end, " $\frac{50}{100}$ " on the crease, and " $\frac{100}{100}$ " under the 100 on the right end. To the side of each label, have them write the decimal represented: 0, .5, and 1. Remind them that percent literally means "out of a hundred" so that writing the percent from the fraction for each is easy. Directly under each label, have the students write the equivalent percent: 0%, 50%, and 100%. Now have students start the association with the first side of the strip: zero is zero as a fraction, decimal or percent. One-half can be expressed many ways: $\frac{1}{2}$ or $\frac{50}{100}$ or .5 or 50%.
11. Continue this process until the students have completed all the fractions on the first side of the strip. Explain to the students that they have now completed a fraction number line on one side of their meter strip and a decimal and percent equivalency line on the other side.

Part II

1. Organize the students into small groups, and give each group 100 similar objects, e.g. colored beads, an assortment of buttons, colored marshmallows, M & Ms, small screws and nuts, tricolor dry pasta noodles, a collection of different coins, or different colored marbles. You might wish to place more than 100 objects on the table and ask the students to count out the 100 and store the rest.
2. Give each group a copy of the handout “Who Has 100 Things?.” Ask each group to sort their 100 objects into at least three groups and then complete the chart and answer the question on the recording sheet. For example, one collection of 100 beads might consist of 25 red beads, 60 green beads, and 15 blue beads and should be recorded as follows:

	Title of Each Group	Number of Items in Group	Fraction	Decimal	Percent
Group #1	Red Beads	25	$\frac{25}{100}$.25	25%
Group #2	Green Beads	60	$\frac{60}{100}$.60	60%
Group #3	Blue Beads	15	$\frac{15}{100}$.15	15%
TOTAL:		100	$\frac{100}{100}$	1.00	100%

3. Have each group present their data to the class on a transparency of the handout “Who Has 100 Things? Recording Sheet.”
4. Encourage students to make the connection between representing their data in the chart or table and representing it graphically in a “rough” circle graph. As they do this, be certain that they understand that this is possible because they know what percent (or part) of the whole is represented by each group.
5. At each table, ask the students to outline the circumference of a circle with their 100 objects, making sure to keep each group of objects together. For example, using the 100 beads, they would begin the circle with the 25 red beads forming one arc of the circle, continue with the 60 green beads connected to the last red bead, and finish with the 15 blue beads completing the circle.
6. Have each group move from table to table to see the other groups’ “circle graphs” that have been created. If the circle graphs have been made on a piece of chart paper, each group can label their graph appropriately and include a creative title.

Classroom Assessment

As students work, circulate and watch carefully as they follow instructions. Answer any questions, and clarify any procedures that may be problematic.

Follow-up/Extensions

Extensions for Part II

1. Discuss how the “rough” circle graphs can be transformed into exact circle graphs by knowing how many degrees are in each wedge or section. Ask students how to determine the number of degrees in each wedge. There are many methods for doing this. One is to set up and solve a proportion. Another is to multiply each percent times 360° . In the bead example, 25% of 360° is 90° , 60% of 360° is 216° , and 15% of 360° is 54° . Have the groups of students calculate the degrees needed for each wedge in their circle graph and use a protractor to draw an exact circle graph.
2. Distribute copies of the handout “Mosaic Art,” and assign the “Mosaic Art” project. Ask the students to display their mosaic art picture and chart at the next class session

Who Has 100 Things? Recording Sheet

1. Sort your 100 objects into at least three groups.
2. Fill in the chart, and answer the question.

	Title of Each Group	Number of Items in Group	Fraction	Decimal	Percent
Group #1					
Group #2					
Group #3					
Group #4					
Group #5					
Group #6					
Group #7					
TOTAL:					

3. How can you find the percent in group #2 without counting?

Mosaic Art

1. Make a picture that uses exactly 100 pieces of different colored paper. (The pieces of paper do not have to be the same size.)
2. Fill in the chart and attach it to your picture. If you use more colors, just add more rows to your chart.
3. Display your mosaic art at the next class meeting.

Color	Number of Pieces That Same Color	Fraction	Decimal	Percent