Square Circles

This lesson allows students to use a variety of units when measuring the side length and perimeter of squares and the diameter and circumference of circles. From these measurements, students will discover the constant ratio of 1:4 for all squares and the ratio of 1:3.14 for all circles.

Learning Objectives

- Identify various units of measure based on their appropriateness for each shape and size.
- Draw conclusions about the relationship of side/perimeter in squares and diameter/circumference in circles based on collected data.
- Through physical representations, develop the idea of a constant that relates a circle's diameter and circumference, namely pi.

Materials

- What Changes, What Stays the Same? Activity Sheet
- What Changes, What Stays the Same Overheads
- Rulers
- Calculators
- Alternate units of measure, such as:
- Pennies
- Paper clips
- Lined paper (use the distance between lines as 1 unit)
- String (can mark inches or cm with a pencil on the string)

Instructional Plan

Student will work in a pairs center activity to complete the What Changes, What Stays the Same activity sheet. These will be collected for future use.

Once all students have filled in the activity sheet, the class can use this data to complete the first page of the overhead What Changes, What Stays the Same. The teacher can record the findings of the student pairs. Students may be given copies of this, too, if the teacher wishes for each student to have a copy of the table. At this point, the teacher should lead the students into identifying a relationship between a square's perimeter and its side. Many students will know that Perimeter = 4 × Side, or P = 4s, but try to get students to think of the 4 as a constant that is equal to P ÷ s. Label the third column of the overhead Perimeter ÷ Side, and calculate the constant for each square. It is important for students to see that this relationship is the same regardless of the square's size or unit of measure, which makes it a "constant." A discussion of constant versus variable may be necessary here.

Now gather the data for the circle in a similar manner and record results on the second page of the overhead. The relationship for the side and perimeter of a square is somewhat obvious since the sides combine to form the perimeter. However, for circles, it is not clear that the diameter contributes directly to the circumference other than a longer diameter

results in a larger circumference. Ask the students to go along with your investigation of whether there might be a similar constant for this situation and label the third column of this Overhead Circumference ÷ Diameter. Ask the students to calculate this ratio for each set of data. Once students are convinced that all values are similar and there might be a constant, the name of this constant (π) and a better approximation can be given. Students can also measure the diameter and circumference of other circles in the room if there is still a question about the existence of this constant.

It is very important that students see that pi is a constant rather than a variable. Its value does not change regardless of the size of the circle or units of measure, and pi always represents the same number. (Students often think of pi as a variable similar to x and y, rather than a constant value like √5.)

Questions for Students

How can we change the formula P = 4s into an equation with P and s on the same side of the equals sign?

[Dividing both sides by s gives $\frac{P}{s} = 4$ which is a relationship that holds for any square.]

Though we may already know P = 4s for squares, why are some of our ratios P ÷ s not coming out to exactly 4?

[Errors in measurement may cause slight errors in the ratio of perimeter to side length. However, all ratios should be close to 4.]

There is a constant that relates a square's side to its perimeter, and there is a constant that relates a circle's diameter to its circumference. Is there a similar constant for a rectangle? Why or why not?

[There is not a constant relationship between a rectangle's perimeter and its width, nor is there a constant relationship between a rectangle's perimeter and its length. However, there is a constant relationship between a rectangle's perimeter and the sum of its width and length. That is, $\frac{P}{w+l} = 2$ for all rectangles. More commonly, this is written as P = 2(w + l).]

Assessment Options

Ask students to write a paragraph expressing the relationship of a circle's diameter to its circumference. The title of the first activity sheet was What Changes, What Stays the Same? Ask students to explain the possible reason this title was used.

Ask students to explain the difference between a variable and a constant.

Allow students to consider the question on the last page of the What Changes, What Stays the Same overheads. Note that students should answer this question without the use of any measuring tools. Ask students to share their answers.

Have students answer the following questions:

If the perimeter of a square is equal to the length of 32 football fields, what is the length of one side of the square? Use the ratio of P/s to set up an equation and solve.

If the circumference of a circle is 112 miles, what is the diameter of the circle? Use the ratio C/d to set up an equation and solve. (This is a good time to show students the pi key on a calculator and how to use it.)

Extensions

Require students to draw several circles on centimeter grid paper. Then, have them determine the radius and approximate area of each circle. By finding the ratio of Area ÷ Radius2, students will again see the appearance of the contstant pi.

Students can create isosceles right triangles of different sizes and measure the lengths of one leg and the hypotenuse. Calculating the ratio of Hypotenuse ÷ Leg for each triangle will lead students to the discovery of the constant relating these two pieces, namely √2.

Teacher Reflection

What were some of the problems students encountered when using the different units of measure?

Are there better items that can be used next time? Which ones worked particularly well?

Were students focused and on task throughout the lesson? If not, what improvements could be made the next time this lesson is used?

How did students demonstrate that they were actively learning?

Were the students led too much in the lesson? Did the students need more guidance?

Did you find it necessary to make any adjustments during the lesson?

Did the materials that the students were using affect classroom behavior or management?

NCTM Standards and Expectations

Measurement 6-8

Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.

Develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes.

This lesson prepared by Kristen Chandler.