PBS Mathline®

*The Middle School Math Project*

# Something Fishy

(Statistics)

## Objective

**Students will estimate the size of a large population by applying the concepts of ratio and proportion through the capture-recapture statistical procedure.**

## Overview of the Lesson

Students are introduced to a problem involving an environmental issue: scientists have determined that the number of fish in the Chesapeake Bay has decreased. Assuming that this is true, scientists must have counted the number of fish and noted the change. How did they count the fish? Groups are formed to explore a wide range of possible strategies. Students are introduced to the capture-recapture method which involves capturing fish, tagging them, returning them and recapturing another sample. In groups, students are given containers (the bay) and an unknown number of fish-shaped cheese crackers to use in simulating the capture-recapture method. Students net the fish-shaped cheese crackers and substitute fish-shaped pretzels to simulate tagged fish. Groups then take samples and record the number of tagged fish and the total number of fish. After several samples are taken, results from each group are recorded. Students apply their knowledge of proportion to arrive at the total fish population.

## Materials

**Each group:**

❶ 1 large container

❷ 300-350 fish-shaped cheese crackers*

❸ 30-40 fish-shaped pretzel crackers*

❹ 1 small aquarium fish net

❺ 1 plate

***Note:*** *Beans, counters, or macaroni can be used in place of the crackers (in which case your students can mark their tagged fish with a pen instead of replacing them with pretzels). Prior to the lesson, place the same number of goldfish crackers in each of the containers. Prepare a cup containing pretzel fish for each group.*

## Procedure

Introduce the problem. Ask students what the environmentalists would need to know to determine that the fish population has decreased. *They must have determined the fish population at least twice.*

Place students into groups. Facilitate the groups while they discuss strategies for determining the number of fish in the bay. Allow the groups to share ideas with the entire class. (It is okay if students do not come up with the capture-recapture method during their discussions.)

After students have had a chance to discuss their ideas, identify the capture-recapture procedure as a statistical method often used to estimate large populations of animals in the wild. Connect the ideas presented by students during their discussion to the concepts that underlie the capture-recapture procedure.

The capture-recapture method is based on the idea that if we measure a portion of a large population, count, tag, release them, and then take another sample, we should be able to estimate the total population based on the proportion of number tagged to the total sample. *(This method is dependent upon the assumption that the population will be randomly distributed throughout the sample — like fish in a bay.)*

Since it is unrealistic for your class to capture fish in the bay, introduce the idea of a simulation. Explain to students that they are going to use a container to represent the bay, and crackers to represent the fish in the bay.

**CAPTURE**

1. Capture a sample of goldfish from the bay (container) using the net and count them.
2. Tag these captured fish by replacing each one with a pretzel goldfish. *(Note: Since in this lesson we are tagging by replacement, the goldfish replaced by pretzels can no longer count as part of the population and MUST be disregarded.)*
3. Put these tagged fish back into the bay. *(Emphasize that the number in the population is unchanged.)*

**RECAPTURE**

4. Since fish crackers don't swim, mix the fish to distribute the tagged fish.
5. Capture another sample from the bay using the net.
6. Record the total number of fish in this sample and the number of tagged fish in this sample. Return the entire sample to the bay.
7. Repeat steps 4 - 6 as many times as time allows.

*After completing the capture portion of the capture-recapture method, ask the following:*

☛ What information has been obtained from the capture? *(How many fish are tagged.)*

☛ Do we know how many are in the bay yet? *(No.)*

Instruct students to use a ratio to show how the number of tagged fish relate to the total number of fish in the bay. Place students' ratios on the board and have the class reach a consensus on the one to use. It should be similar to the one below.

***Note:*** *Remember that it is also correct to use the reciprocal, as long as there is consistency in the proportion.*

$$\frac{\text{NUMBER TAGGED}}{\text{TOTAL NUMBER IN BAY}}$$

*Next, ask students:*

☛ When the fish are recaptured, what could be found in the sample? *(All tagged, none tagged, some tagged and some not tagged.)*

☛ What do you expect to find? *(Some tagged and some not tagged.)*

☛ After the first *recapture*, what information will you obtain? *(How many total fish are in the recapture and how many of them are tagged.)*

Now, have students use a ratio to show how the number of tagged fish relate to the total number of fish in the bay. Put ratio on the board or overhead next to the first one.

$$\frac{\text{NUMBER TAGGED IN RECAPTURE}}{\text{TOTAL NUMBER IN RECAPTURE}}$$

Be sure that students understand the relationship of a given ratio to a given proportion. Remember that the capture-recapture method is based on the assumption that these two ratios are equal; thus, the two ratios can be set up as a proportion.

$$\frac{\text{NUMBER TAGGED}}{\text{TOTAL NUMBER IN BAY}} = \frac{\text{NUMBER TAGGED IN RECAPTURE}}{\text{TOTAL NUMBER IN RECAPTURE}}$$

Review the part of the proportion which the students already know (the total number of tagged fish) and the parts of the proportion which they find from their recapture procedure (the number of tagged fish in the recapture and the total number of fish in the recapture). Have groups begin the recapture portion of the capture-recapture method. If possible, each group member should have an opportunity to recapture a sample.

After all of the data has been collected, each group should construct and solve their proportions. The use of calculators should be permitted. In each group the mean of all of their trials will be used for the group estimate.

Record the estimates from each group. Have the class examine them to discuss the range of estimates and the probable number of fish in the bay.

Finally, tell the students exactly how many fish you placed in each bay. (Each bay should have had the same number of fish.) Students should then compare the actual number of fish to their results.

## Extensions & Connections

*Ask students to discuss the following:*

- Why did the capture-recapture method provide a reasonable estimate?
- Would estimates be closer to the actual if a larger sample is used?
- If you average all of the group averages, would you get the same result as pooling that data, solving the proportion and dividing the result by the total number of bays?

Ask students to explore other situations where the capture-recapture method could be useful (i.e. other animal populations, the value of coins in a jar, etc.). As students make suggestions, they should decide if and why it will or will not work. What conditions make the use of the capture-recapture method possible?

Have students investigate other strategies for estimating the size of large populations.

In the video and the lesson plan, the proportion was set up in a certain way. Have students explore to find other ways the proportion can be presented.

You may wish to show the video entitled, "Sharks" from the *Challenge of the Unknown Film Series (Phillips Petroleum Company, Bartlesville, Oklahoma).*

## Resources

Swetz, Frank and J.S. Hartzler. *Mathematical Modeling.* (1991) National Council of Teachers of Mathematics. Reston, Virginia.

"Sharks" from the *Challenge of the Unknown Film Series (Phillips Petroleum Company, Bartlesville, Oklahoma).*

## Ideas for Online Discussion

(*Some ideas may apply to more than one standard of the* ***NCTM Professional Standards for Teaching Mathematics.)***

### Standard 1: Worthwhile Mathematical Tasks

❶ "In selecting, adapting, or generating mathematical tasks, teachers must base their decisions on three areas of concern: the mathematical content, the students, and the ways in which students learn mathematics." (p. 26) What important mathematical concepts are incorporated in this lesson?

❷ Did the students seem to have command of all the mathematical concepts needed to solve the problem? Discuss whether or not you would use this lesson as an introduction to the concept of proportion.

### Standard 2: Teacher's Role in Discourse

❸ "Well posed questions can simultaneously elicit and extend students' thinking." (p. 35) Were there any questioning techniques used which you felt were particularly good in eliciting thoughtful responses from the students? Share some of the techniques that you use with your students.

❹ How did the teacher challenge her students to think about their reasoning and to then consider other alternatives?

❺ "Decisions about when to let students struggle to make sense of an idea or a problem without direct teacher input, when to ask leading questions, and when to tell students something directly are crucial to orchestrating productive mathematical discourse in the classroom." (p. 36) Comment on the dynamics of the learning which occurs when teachers interact with students in small group settings.

❻ Comment on the level of participation of the students in the video. What techniques do you use to insure that all students are engaged during all phases of a lesson?

### Standard 5: Learning Environment

❼ Cite some strategies that you have used with your students which are designed to ease anxiety and promote free assimilation and expression.